Social Studies 201
September 22, 2003

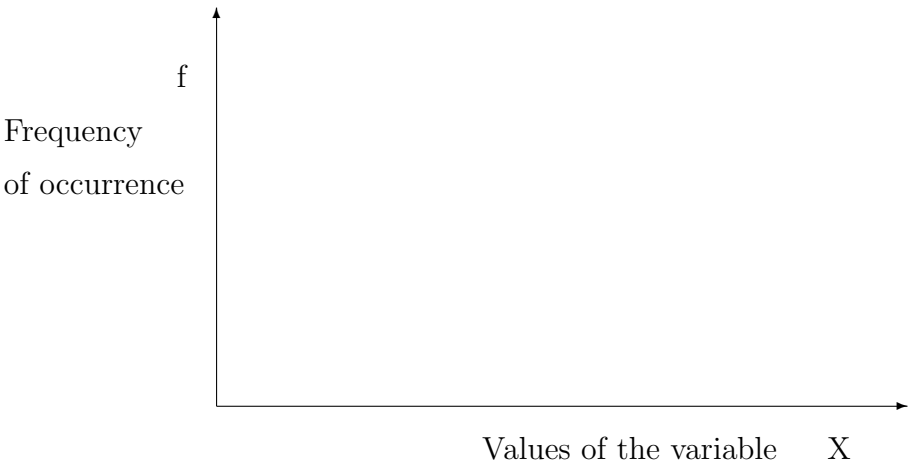
Histograms and Density

1. Introduction

From a frequency or percentage distribution table, a statistical analyst can develop a graphical presentation of the distribution. One common way to do this is with a bar chart or histogram. These notes outline procedures for constructing these.

The conventional statistical approach to presenting a frequency distribution diagram is to place the variable on the horizontal asis and the frequency, or percentage, of cases on the vertical axis. As in Figure 1, the values of the variable (X) are placed along the horizontal axis and the vertical axis represents the number, or percentage, of cases.

Figure 1: Conventional labelling of axes for diagram of frequency distribution



While the axes might be reversed, so frequencies are on the horizontal and values of the variable on the vertical, the convention in most statistical presentations is as in Figure 1.

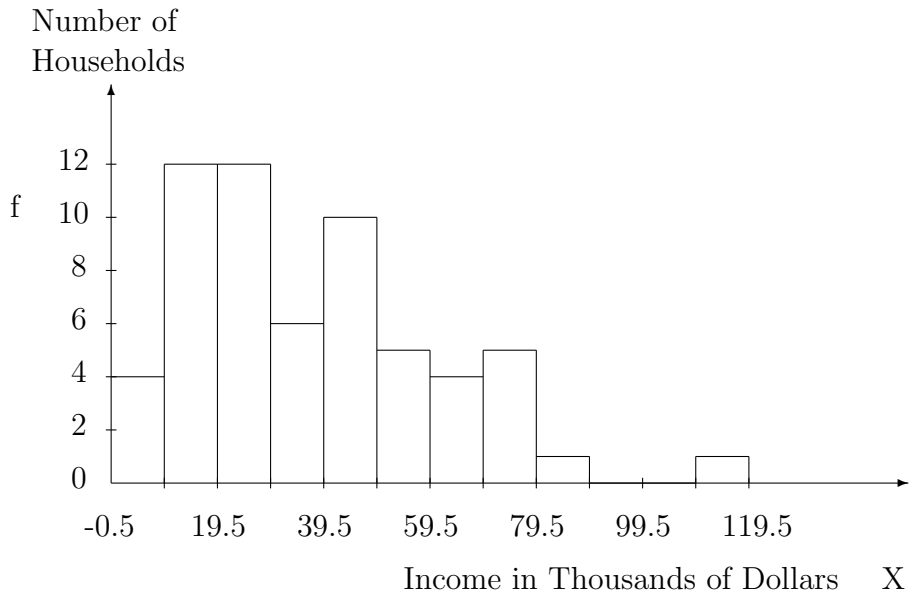
Various forms of line and bar charts can be presented using this format. See section 4.7.1, pp. 121-126, of the text for examples of line and bar charts.

2. Histograms

A histogram is a bar chart where the height of each bar represents the relative frequency of occurrence of the cases. The bar chart of the stem and leaf display of September 16, 2003 is in Figure 2. Here the data have been grouped into intervals of ten thousand dollars, and the height of the bars represent the frequencies of occurence of each category or interval. For example, the bar for an income of 9.5 to 19.5 is 12 units high because there are twelve households with income in this range. For the interval 109.5-119.5, there is only one household, so the bar ia one unit high.

In words, this histogram can be described as follows. There are relatively few households at the lowest income level of less than ten thousand dollars, with the greatest number of households at incomes from 10-29 thousand dollars, and considerable numbers of households with incomes of 30-49 thousand dollars. As income increases, there are relatively fewer households at higher income levels.

Figure 2: Histogram of income distribution for sixty Saskatchewan households, 1994



Intervals of different width

In some cases it may make sense to reduce the number of intervals into which the data are grouped. In the above example, there are relatively few households with incomes of eighty thousand or more dollars, but the intervals contiue to 110-119. In some data sets, there may be values even greater than this, with the result that there would need to be many intervals to account for all possible values. In order to handle such situations, it may make sense to collapse the intervals, so there are fewer, but wider, intervals for values that occur infrequently.

As an example of this, In Table 1, the household incomes from the stem and leaf display are grouped into intervals of ten thousand dollars each for the range of incomes up to fifty thousand dollars. But for higher incomes, the cases are grouped into wider intervals. That is, in Table 1 the intervals of 50-59 and 60-69 are combined into a single interval from 50 to 69 with nine households. The nine cases in this interval come from 4 households with income of 50-59 and 5 households with income of 60-69. Similarly, the intervals of 70-79, 80-89, and 90-99 have been grouped together into the 70-99 interval, and the highest incomes have been grouped into the interval 100-119.

Table 1: Distribution of income, sixty Saskatchewan households, 1994

Income in Thousands of Dollars (X)	Real Class Limits	f
0-9	-0.5-9.5	4
10-19	9.5-19.5	12
20-29	19.5-29.5	12
30-39	29.5-39.5	6
40-49	39.5-49.5	10
50-69	49.5-69.5	9
70-99	69.5-99.5	6
100-119	99.5-119.5	1
Total		$n = 60$

Using these new, wider intervals makes the presentation of the data more efficient, in that there are fewer intervals in the table. When presenting the data from Table 1 graphically, it is necessary to compensate for the different interval widths. In order to provide an accurate presentation of the distribution, the density or relative frequency of occurrence of the number of cases in each interval should be calculated. This is done as follows.

Density

The density of occurrence of a variable is the number of cases per unit of each value of the variable. For each interval, the density, or relative frequency, is:

$$\text{Density} = \frac{\text{frequency}}{\text{interval width}}$$

The idea of obtaining the density is to adjust for the different width of the intervals, so the frequency of occurrence of the variable per unit of the variable is presented consistently across the different values of the variable. Table 2 provides a calculation of the densities for the grouping in Table 1.

Table 2: Distribution of income, densities or relative frequency of occurrence

Income in Thousands of Dollars (X)	Real Class Limits	width (w)	f	Density=f/w
0-9	-0.5-9.5	10	4	4/10=0.40
10-19	9.5-19.5	10	12	12/10=1.20
20-29	19.5-29.5	10	12	12/10=1.20
30-39	29.5-39.5	10	6	6/10=0.60
40-49	39.5-49.5	10	10	10/10=0.10
50-69	49.5-69.5	20	9	9/20=0.45
70-99	69.5-99.5	30	6	6/30=0.20
100-119	99.5-119.5	20	1	1/20=0.05
Total		$n = 60$		

By dividing the frequency of occurrence by the interval width, the densities represent the relative frequencies of occurrence of the variable, after taking account of interval width. For example, in the first interval from 0 to 9 thousand dollars, there are 4 cases spread across 10 units of the variable, so the density is $4/10 = 0.40$. In the case of the 70-99 thousand dollar interval, there are 6 cases, but they are spread across thirty thousand dollars of income. In this case, the density is $6/30 = 0.20$. That is, the density or relative frequency of occurrence, for the 0-9 interval is double that for the 70-99 interval. In this case there are relatively more households in the lower than in the higher interval.

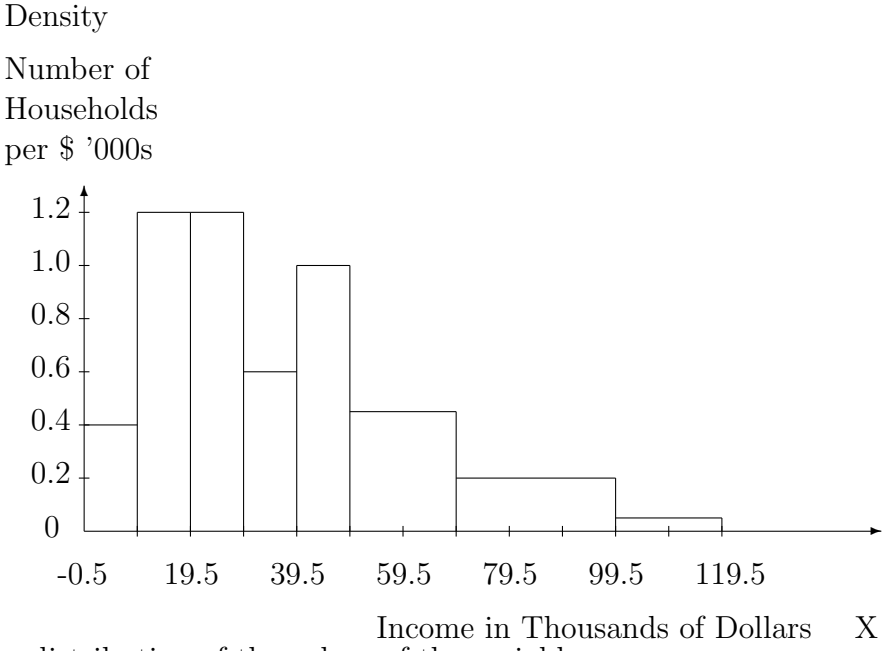
These densities are especially useful when graphing the distribution from such a table as a histogram. If the densities are used, the bars of the histogram accurately represent the relative frequencies of occurrence of the variable. For the frequency distribution of Table 2, Figure 3 presents the histogram. Note that the horizontal axis is the same as in the case of Figure 2, but that the vertical axis is labelled with the densities of occurrence. If the densities had not been used, the diagram would be inaccurate, overrepresenting wider intervals. But by calculating densities, each bar is the proper height, representing the relative frequency of occurrence of cases for each interval.

Figure 3 contains the histogram of income distribution for the sample of sixty households, where incomes are grouped into the intervals of Table 2. Comparing Figures 2 and 3, it can be seen that the histograms have the same general shape, even with intervals of different width. This is the way such a diagram of a frequency distribution should be – that is, the basic shape of the curve should not change just because the data were grouped differently. The only real difference is that the grouping of Table 2 does not have quite as much detail as the grouping of Table 1. Asa result, the bars on the right end of Figure 3 are wider. The height of these bars though is roughly the average height of the bars of the corresponding intervals of Figure 2.

3. Conclusion

If all intervals for a variable X are of the same interval width, then the histogram is constructed with bars equal in height to the frequency of occurrence of each interval. But where the intervals into which data are grouped have different interval width, it is best to calculate and graph the densities on the vertical axis. This presents a more accurate representation of the actual

Figure 3: Histogram of income distribution for sixty Saskatchewan households, 1994



distribution of the values of the variable.

One other use for densities is the calculation of the mode. While this is covered in more detail in Chapter 5, the mode, or most common value, is the value of the variable where the histogram reaches its peak. In order to accurately determine this, when intervals are of unequal size, densities can be used to determine where the peak of the histogram occurs.