Social Studies 201

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Measurement of variables

See text, chapter 3, pp. 61-86.

This first part of Module 2 and Chapter 3 of the text examine ways of classifying and measuring variables to describe members of a sample or population. Characteristics of a population (the variables) can be organized and measured in various ways. These notes discuss the different possible ways of measuring variables, and the advantages and disadvantages of each.

Four scales of measurement are discussed in these notes – the nominal, ordinal, interval, and ratio scales of measurement. The meaning of these forms of measurement and the distinction among them is basic to working properly with quantitative data. Once you have completed this part of Module 2 you should be able to recognize the different forms of measurement, understand the distinctions among them, and have an idea of what are proper and improper applications of each. An introductory example follows.

> **Example 2.1 – Sex, attitude, and age**. As an example of how forms of measurement differ, consider three variables used to describe members of a population – sex, attitude, and age. The variable sex has only two categories, male and female, and ordinarily numerical values are not associated with the categories male and female – these are merely the names we give to the two biological categories of sex.
>
> In contrast to sex, the age of an individual is ordinarily measured numerically in number of years, so that a student may be 20 years old and a professor 60 years old.
>
> When measuring attitudes about a social or political issue, a scale from strongly disagree to strongly agree is often used. This measurement may be constructed in numerical form, on a five-point scale from 1, indicating strongly disagree, to 5, indicating strongly agree. While the numbers attached are somewhat arbitrary, one key characteristic of such an attitude scale is that the values from strongly disagree to strongly agree are ordered or ranked, so an

individual response can be compared as being more than, less than, or equal to responses of others.

In this example, each of the three variables has a different form of measurement. Sex is measured using a nominal scale of measurement, a classification or categorization; attitude has an ordinal level of measurement, that is the values associated with attitude are ordered or ranked; and age has an interval and ratio scale of measurement, with meaningful measures of distances and ratios between values.

Each of the above forms of measurement will be defined in the notes that follow. Before defining and providing examples of each, these notes contain a few preliminary comments.

Hierarchical.

The four forms of measurement are hierarchical in that each higher level of measurement has all the characteristics of the lower levels of measurement, but some added characteristics as well. The hierarchical structure is as follows.

Level	**Name**	**Characteristics**
highest	ratio	categorization, ordering, distance, and ratio
	interval	categorization, ordering, and distances
	ordinal	categorization and ordering
lowest	nominal	categorization only

The ratio scale is the highest or strongest level of measurement and the nominal scale is the lowest or weakest level of measurement. An ordinal measurement is a stronger form than a measurement that is no more than nominal; an interval measurement is s stronger form than an ordinal measurement; and the ratio level is the strongest form of measurement.

In this hierarchy, all forms of measurement are at least nominal, but some are ordinal, in the sense that values of the variable can be ordered relative to each other. In addition to being nominal and ordinal, some types of measurement permit meaningful measurment of the distance or interval between values. Finally, variables that are nominal, ordinal, and interval usually have a ratio level of measurement, permitting meaningful calculation of ratios between values of a variable.

As an example of different ways that a researcher can ask survey questions, consider the following example of ways of determining political views of individuals. This demonstrates the different types of measurement and the hierarchical quality.

Example 2.2 – Questions about political issues. A researcher investigating political views of a sample or population will ask individuals a variety of questions. The following list contains three possible questions, the variable that is produced from each question, and the level of measurement of each variable.

Survey question	**Variable**	**Level of measurement**
Which political party do you support?	Political party supported	Nominal only – classification of parties
How strongly do you support the revolutionary party?	Strength of support for party	Ordinal – ordering or ranking of support
What percentage of votes will the revolutionary party obtain?	Anticipated percentage of vote	Interval and ratio – meaningful distances ratios between values

In this example, each question provides a way of measuring a variable concerning political issues. The first question produces a variable that does no more than categorize people into groups by party supported, a nominal level only.

The second question is like an attitude or opinion question, asking respondents to state how strongly they support the revolutionary party. This produces a categorization of people into different levels of support, but these categories are ordered, from no or minimal support, to strong or complete support.

The third question asks respondents to provide an estimate of the percentage of votes they expect the revolutionary party to obtain. This classifies respondents into different levels of estimated percentages, these percentages are ordered, and the distance between the percentages can be determined. The anticipated percentage of vote thus has an interval level scale of measurement.

These questions, along the with variables they produce, demonstrate the different levels of measurement and the hierarchical structure of these levels.

Reasons for different forms of measurement. Forms of measurement, or scale, differ across variables for the following two reasons.

1. Different types of phenomena are investigated by a statistical analyst.

2. A statistical analyst uses different questions and ways of defining the concept being measured. These are connected with the theoretical and operational issues of data production, discussed in Module 1 and Chapter 2 of the text.

Some physical concepts such as distance or time, and social science concepts such as income or price have well-defined units of measurement. Distance can be measured in metres, time in hours, and income and price in dollars. Each of the units metre, time, and dollar is well-defined and understood and each unit has mimimal or no variation across space and time.

Other variables such as university faculty or major, have no well-defined unit of measure, but are no more than classifications or categorizations of individuals into groups or categories. The situation is even more difficult for variables such as race and ethnicity, where there not only no unit of measurement, but generally agreed upon classification. As a result that different analysts and commentators produce different classifications of race and ethnicity.

Falling between the strong form of measurement of distance and time and the weakest form of measurement of classification, is the ordinal level of measurement. These could be variables such as degree of alienation, extent of agreement with same-sex marriage, or position on the left-right political spectrum. For such variables, values can be ranked or ordered but there is no well-defined unit of measure that different statistical analysts could agree upon. These ordinal variables are particularly useful for the social sciences, permitting researchers to rank people but, to date, social scientists have not been able to develop or agree on what a unit of measure might be for such variables.

Four levels of measurement. In the following notes, each of the four forms of measurement are defined, with examples of each provided. The presentation is in order, from the nominal scale, to the ordinal, the interval, and finally the ratio scale.

Nominal scale of measurement

See text, section 3.2.1, pp. 62-63.

A nominal scale is simply a classification of characteristics or values of a variable into different categories. Since all variables are ways of describing the different categories or values of members of a population with respect to some characteristic, all variables have a nominal scale of measurement. Since each variable provides a means of categorizing or classifying members of a sample or population, all variables are measured at least at the nominal level.

> **Definition of nominal scale**. A nominal scale is a scale that classifies the various values of the variable into categories. These categories have names, or possibly numbers, associated with them.

In the case where numbers are given to values of a nominal scale, the numbers are usually arbitrary and represent no more than codes for classifying and storing the data.

> **Example 2.3 – Variables with nominal scale**. Following are examples of variables with a nominal scale only, that is, these are variables that cannot be measured at more than the nominal level of measurement.
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> - **Species of animals**. The different species of animals (homo sapiens, cats, insects, whales) constitute a classification system for living animals. Each animal is a member of one species and only one species, and members of each species differ in characteristics from those of other species. Such a variable is not quantitative in that there is no obvious numerical or quantitative value to be associated with each species – this is merely a categorization of animal types.
> - **Sex of individuals**. These are two different biological characteristics of humans, male and female. While some may consider one sex to have more of some characteristic than another (e.g. males may be considered more aggressive, females more nurturing), sex itself has no numerical or other

qualities that make one sex more or less than the other. As a result, sex is a variable that has no more than a nominal scale of measurement.

- **Religion**. There are many different religions, but again these are just different ways individuals categorize themselves. Consequently, religion has a nominal scale of measurement and no more than a nominal scale.
- **Ethnicity, nationality, race**. In terms of how classification occurs, this is similar to religion, with each individual identifying himself or herself as having a particular ethnicity, nationality, or race. Again, these are only nominal scales. There may be disagreement or confusion about the meaning of such classifications, but any such classification of people by ethnicity, nationality, or race is no more than a nominal from of measurement.
- **Political party preference**. Again, this is merely a nominal scale a classification of people into different categories depending on which political party they prefer to support, or not support.

Additional notes on nominal scales

1. Coding of nominal scales

Categories associated with a nominal scale are usually given names – for example male and female for sex, or Presbyterian, Buddhist, Doukhobour, etc. for religion. Numbers can be attached to the categories for each nominal scale, but these numbers are arbitrary and used only for convenience. For example, for the variable sex, each category can be given a number, such as 1 for male and 2 for female. But these numbers are arbitrary, in that females could be coded as 1 and males as 2. Such coding is often used when working with computer programs to analyze data – each characteristic of a variable is given a numerical value so it can be entered into a computer data set, stored on the computer, and used when required. Do not attempt to conduct arithmetical operations using these codes – they are merely for purposes of classification, storage, and further organization of the data.

> **Example 2.4 – Codes for University of Regina Faculties**. In the SSAE98 data set, the variable "Faculty" has no more than a nominal scale of measurement. Yet each faculty has a numerical value associated with it – Administration is 1, Arts is 2, Education is 3, and so on. These numbers are arbitrary and are assigned as a result of placing the faculties in alphabetical order. If the list had been organized in some other manner, the number associated with each Faculty would be different. The numbers are attached to the Faculties only for purposes of keeping track of each category and for entry into a computer data set.

2. Exhaustive and mutually exclusive

When constructing nominal scales, it is preferable for the researcher to construct the scales so the categories of each variable are exhaustive of all possible characteristics and mutually exclusive of each other.

- **Exhaustive**. If the categories for a variable are exhaustive, each individual will be in at least one of the categories of a variable. For a variable such as sex, there are only two categories, male and female, so an exhaustive set of categories is easily constructed.

For political party preference, there may be a number of minor parties and some respondents may not prefer any party. In this case, the variable "political party preference" can be constructed with an exhaustive set of categories by listing all possible political parties and including a category for "none of the parties." In addition, an "Other" category can be added, to include any parties were missed in the initial listing and any other possible unanticipated responses.

When constructing variables, by including "none" and "other" as possible responses, each individual can be classified into some category, even if the category is no more than none or other. In addition, when entering data into a computer data set, categories such as "no response," "refused," or "missed" might be included. That is, every individual is assigned some category, so the variable is exhaustive of all possibilities.

- **Mutually exclusive**. By "mutually exclusive" a statistical analyst means categories that do not overlap, so an individual would not be included in two different categories at the same time. That is, for a variable with a mutually exclusive set of categories, these categories do not overlap with each other. For example, for sex we ordinarily consider male and female to be mutually exclusive categories. For attitude questions, it is ordinarily expected that responses such as strongly disagree, somewhat disagree, neutral, somewhat agree, and strongly agree are mutually exclusive. That is, a statistical analyst would not expect an individual to be simultaneously in agreement with a statement about a political issue and neutral on the same issue. Each individual response is expected to be in only one of these categories. For purposes of statistical analysis, whenever possible, it is advisable to construct variables so the categories are mutually exclusive.

  An example of a nominal scale that does not have mutually exclusive categories is education of respondent, as measured by educational institution attended. Education is often measured on the following list: primary, secondary, post-secondary, university, technical school, business school. In this list, university and post-secondary overlap and are not mutually exclusive. In addition, a respondent may have attended several of these types of school. While education measured by such categories may be a useful variable, because of the overlapping categories of classification, it is not so readily analyzed, since an individual may state they have attended more than one of these levels of educational institutions. A statistical analyst might thus ask a second question, requesting respondents to state how many years of education they have. This additional variable is useful, producing a mutually exclusive set of years. The two questions on education give the analyst a more complete idea of the type and level of education of each respondent.

The mutually exclusive criterion of classification is often violated when measuring ethnicity. Individuals may have a mixed ethnic ancestry, for example, an individual might be of Russian, French, and Mexican ancestry. While the different categories of ethnicity may be mutually exclusive of others, an individual's ethnicity may include several ethnic categories, so that individual ethnicity is not easily measured on a single scale of ethnicity.

The following sections of these notes introduce variables that have a nominal scale but are more than nominal – that is, all scales of measurement for variables are nominal, but some are more than this.

Ordinal scale of measurement

See text, section 3.2.1, pp. 64-65.

Values of a variable with an ordinal scale of measurement can be ranked or ordered with respect to each other. This is considered a higher or stronger level of measurement than the preceding form of measurement, where the variables had no more than a nominal scale of measurement. More and stronger forms of statistical analysis can be performed using variables where the characteristics can be ranked or ordered with respect to each other.

> **Definition of ordinal scale**. An ordinal scale is a scale that is nominal, and one in which each value of the variable can be ordered, or ranked, as more than, less than, or equal to any other value of the variable.

Associated with an ordinal scale are the words "more than," "greater than," "less than," and "equal to." That is, for a characteristic that is measured using an ordinal scale, any member of a population has more than, less than, or equal amounts of this characteristic, when compared with any other member of the population. Alternatively stated, using an ordinal scale, each person can be ordered with respect to any other person, with respect to that variable.

In mathematical notation, each value of an ordinal scale can be compared to other values of the variable using one of the following mathematical operators:

$<$ less than

$=$ equal to

$>$ more than or greater than

The concept of ordinality involves ordering or ranking people on some characteristic. Variables such as sex or ethnicity do not lead to any such ranking; examples of variables that can be ordered or ranked are as follows.

Example 2.5 – Examples of ordinal variables.

- **Order of finish in a race or competition**. In a race or competition, contestants are often ranked by whether they finish first, second, third, and so on. The distance between the rankings may not be important. For example, when gambling on a horse race, in terms of payoff all that matters is whether the horse you selected finished first (win), second (place), or third (show). These rankings constitute an ordinal scale for the variable "finish of the race."
- **Attitudes or opinions**. Most attitude and opinion scales used in the social sciences are ordinal scales. That is, individuals are asked to state their level of agreement or disagreement on some statement about a political or social issue. The format followed for responses to such statements is often as follows

View	**Code**
Strongly agree (SA)	5
Somewhat agree (A)	4
Neutral (N)	3
Somewhat disagree (D)	2
Strongly disagree (SD)	1

Such a list constitutes an ordering or ranking of views as less than, equal to, or more than others in terms of extent of agreement or disagreement. That is, anyone who responds 4 is more in agreement than someone who responds with a 2. Two individuals that give the same numerical response, say a 2, are equal to each other in terms of agreement or disagreement on this issue, but are less in agreement than someone who responds with a 3.

There are many modifications of such an attitude scale. For example, some researchers prefer only a four-point scale, with the neutral category eliminate. This forces people to state extent of agreement or disagreement. Alternatively, some researchers prefer a seven-point scale. Respondents are asked to state their extent of agreement or disagreement with a statement on a seven-point scale where 1 indicates strong or total disagreemeent, 7 indicates strong or total agreement, and 4 represents a neutral or middle view.

- **Social class**. If a researcher adopts a stratification approach to the measurement of social class of members of a population, the following ordinal scale might be used.

Class	**Code**
upper class	5
upper-middle class	4
middle class	3
lower-middle class	2
lower class	1

Those nearer the top of such a scale are considered to have greater status, prestige, income, or wealth. Those lower on the scale are considered to have less of one or all of these. This produces a ranking of individuals according to their "social class."

Other measures of social class such as socioeconomic status or prestige, including the Canadian measures of social class structure designed by Blishen, Pineo, and Porter, have at least an ordinal scale of measurement. Examples of these will be used in the notes that follow.

A Marxian definition of social class, using the categoies such as bourgeoisie, proletariat, landlord, petty bourgeoisie, and lumpenproletariat is not truly ordinal. The manner in which the Marxian categories of class are defined is with respect to relations of production, rather than using a ranking on specific variables.

- **Religiosity**. A researcher might attempt to measure the religiosity of members of a population by asking them how often they attend religious services. An ordinal scale for religiosity might be constructed as follows, going from more religious to less religious.

Attend daily
Attend several times weekly
Attend weekly
Attend several times monthly
Attend once a month
Attend several times a year
Attend once a year
Never attend

In this case, numbers need not be attached. That is, someone who responds “attend weekly” has a greater religiosity (at least as measured by this scale) than does someone who responds “attend once a month.” However, numbers from 1 to 8 might be attached to the categories in order to assist with data analysis.

Additional notes on ordinal scales

1. Ordinal scales and numerical values

Numbers can be attached to different values of an ordinal scale, but they need not be. A question concerning safety on campus was asked in the SSAE98. The question is "How much of a problem is personal safety on campus?" and the responses are to be "A great problem," "A minor problem," or "Not a problem." These are ordered or ranked responses even without any numbers being attached. That is, anyone who says indicates that safety is a minor problem is lower on the scale of the variable "Personal safety" than is someone who responds that it is "A great problem." In the survey, numbers were attached for purposes of data entry, but in order to obtain a measure of "Personal safety" they would not have needed to be attached.

2. Uses of an ordinal scale

When analyzing data, it is useful to have variables measured on at least an ordinal scale. Instead of merely sorting members of a population into different categories, as is the case with a scale that is no more than nominal, an ordinal scale allows a researcher to say something about who has more or less of the characteristic being studied. This permits a more sophisticated form of analysis, where a researcher can begin to investigate how and why some are ranked higher than are others with respect to the phenomenon being examined.

When a variable has an ordinal scale of measurement, statistical measures such as percentiles, quartiles, and the median can be calculated for the variable. We will examine these measures Module 3 and in Chapter 5 of the text.

Interval scale of measurement

See text, section 3.2.3, pp. 66-69.

Variables with an interval scale of measurement have a well-defined unit of measure, so that distances between values of a variable can be meaningfully measured. Examples are height or weight where the metre or the kilogram, respectively, are well-defined units of measurement that can be used to accurately measure differences in heights or weights of different people.

> **Definition of interval scale**. An interval scale is a scale that is nominal and ordinal, and one in which equal numerical differences represent equal quantities or magnitudes of the characteristic.

As an example, consider any measurement in dollars where, at any point in time, one dollar is equal to any other dollar. If I have $4 and you have $3, this is a difference of $1. Similarly, if someone has $2 and another person has $1, that is a difference of $1. Each of these differences of $1 represents exactly the same quantity of dollars, that is, one dollar. While measurements in dollars have a nominal and ordinal level of measurement, they also have an interval levels of measurement, since equal dollar differences represent equal differences. Any variable that can be measured in dollars has an interval scale. The reason is that the dollar is a well-defined and understood unit, and each dollar really represents the same magnitude in monetary terms.

In contrast, while numerical values may be associated with ordinal scales, there is no assurance that the same numerical difference represents the same quantity of the characteristic. Consider an attitude question measured on a 1-5 scale, from 1 meaning strongly disagree to 5 meaning strongly agree. Does the difference between an attitude response of 1 and 2 (strongly disagree to mildly disagree) represent the same difference of attitude as the difference between 3 and 4 (neutral to mildly agree)? Perhaps, but while the difference has a numerical value of 1, it is not clear that the two values of one ($2-1=1$ and $4-3=1$) represent equal quantities of differences in attitude in any well-defined way. For an ordinal attitude scale, the problem is that there is not a unit of measure, and it is not clear how a unit of attitude might be defined.

One of the consequences of having an interval scale of measurement is that values of the variable can be added and subtracted, so that these ordinary arithmetic operations on values of the variable are meaningful. In contrast, for ordinal scales such as attitudes, it may make little sense to add or subtract values of the variable.

Example 2.6 – Examples of interval scales

- **Age**. Age is usually measured as the number of years since date of birth. This produces an interval scale of measurement for age, with the unit being "one year." The measure of age is in units of time, and time has a well-defined and generally agreed upon unit of measure – the second, minute, or hour. These units of time are constant across geographic boundaries and at different times. Similarly, any other variables measured in these same units of time has an interval scale of measurement.
- **Grades**. Class grades in the unit of one per cent (1%) are usually considered to have an interval scale of measurement. While there is a subjective aspect to the grading, most instructors attempt to grade assignments a and examinations as objectively as possible. Percentage points are added for each assignment or examination to compute a total grade for a class. If someone scores 76% for the class, this is 5 percentage points more than someone who scores 71%. If you obtain a grade of 69%, this means you would need to obtain one more percentage point to produce a grade of 70%. Treating grades as interval level measurements also means that the grades for different classes can be added and a grade point average can be meaningfully calculated.
- **Religiosity**. In the examples of ordinal variables, the relative number of times that an individual attended religious services was treated as an ordinal scale of measurement. By changing the definition of the variable, defining it as the number of times per year that an individual attends religious services, religiosity could be constructed as an interval scale. Each attendance at a religious service is one unit, and the total number of times attended per year could be counted to produce an interval scale. This demonstrates that the scale of measurement depends in part on the structure of the question and the potential responses provided to respondents. While variables such as sex or ethnicity cannot

be constructed as ordinal or interval scales, some phenomena can be defined in a way so they are measured as either ordinal or interval scales. For example, amount of education might be defined as highest level completed (ordinal level measurement) or years of education (interval level measurement).

- **Income**. This is ordinarily measured in dollars, a well-defined unit of measure, so income has an interval scale of measurement.

Additional notes on interval scales

1. Unit

If a well-defined unit of measure is available, meaningful to everyone, and constant across time and space, then the scale of measurement is at least an interval scale.

2. Scales may be ordinal only but treated as interval

In the social sciences there are many variables that statistical analysts commonly treat as having an interval level of measurement, even though they probably have no more than an ordinal level of measurement. Some examples follow.

- **IQ**. Intelligence quotient (IQ) is often treated as having an interval scale. For example, someone with an IQ of 115 may be regarded as having an IQ that is 10 points less than someone with an IQ of 125. While statistical analysis of IQ may take this form, it is not clear what a unit of IQ might mean. As a result, some analysts treat IQ as having only an ordinal scale, ranking individuals by level of intelligence. But exactly how much more or less IQ an individual has, compared with any other individual, is not so clear. Other analysts have less hesitation about treating IQ as an interval scale – these analysts might add IQ scores and average them across a number of individuals.

- **Socioeconomic status (SES)**. A similar problem is associated with social stratification research, where socioeconomic status (SES) is sometimes used. In Canada, the Blishen or Pineo-Porter indexes may be used to rank occupational prestige or status. Each occupation is given an SES, with large values representing occupations with greater prestige, status, and income; occupations with lower SES have less prestige, status, and income. For example, in the Blishen SES rating, a judge has a score of 93.3 while cashiers and tellers have a score of only 28.3. Such ratings may be developed by asking people to rank the status or prestige of occupations, or they may be derived from income and education levels of those filling these occupations. While the ranking of SES makes some sense, as in the case of IQ, it is not clear what the unit of SES is. Again, some analysts treat such scales as ordinal while other researchers consider them to be interval scales. The Blishen score is a combination of income and years of education, so each of its components has an interval level scale of measurement. This may make it appropriate to treat Blishen scores as having an interval levelof measurement.

- **Attitudes**. Even though attitudes measured on a 1-5 or 1-7 scale from strongly disagree to strongly agree have no more than an ordinal scale of measurement, it is common for researchers to treat these variables as if they were measured on an interval scale. Average, or mean, attitude may be reported, implying that attitudes of different people can be added and averaged. In order to do this, the statistical analyst is treating attitude as having an interval level scale of measurment. The difficulty associated with this is that the unit of attitude is not well defined. This issue is not usually addressed by analysts adopting this approach.

While it is common for researchers to treat ordinal level variables as having an interval scale of measurement, caution should be exercised when doing this. Strictly speaking, many statistical techniques are appropriate only when using variables having interval level scales. But researchers working with attitudes and similar types of variables commonly treat these variables as if they have interval level scales and use techniques appropriate for the interval level. Some of the consequences of this approach will be examined later in the semester.

3. Uses of interval scales

If a variable is measured at the interval level, researchers can conduct more extensive statistical analysis than when scales have no more than a nominal or ordinal level of measurement. For interval scales, the numerical difference between two values is meaningful and represents a number of units of the variable. Values of interval scales can be added or subtracted, so the mean value of such a variable is meaningful (see Module 3 and Chapter 5 of the text). In addition, many other statistical procedures, such as those discussed in Modules 6-7 and Chapters 8-9 of the text, can be used to analyze particular variables or relationships among variables. Whenever possible, it is advisable for the researcher to define variables so they can be measured at the interval level.

Ratio scale of measurement

See text, section 3.2.4, pp. 69-70.

Most variables measured at the interval level also have a ratio level of measurement. That is, ratios between values are also meaningful, so these variables also take on the ratio scale of measurement. That is, if values of the variable can be added and subtracted (interval scale), then dividing one value by another, to calculate the ratio between these values is usually appropriate.

> **Definition of ratio scale**. A ratio scale of measurement is an interval scale with the additional property that equal ratios between two possible values of a variable represent equal magnitudes. An additional characteristic of a ratio level scale is that the zero point of the scale is not arbitrary, but a value of zero indicates that none of the characteristic is present.

An example of a variable with a ratio level scale is distance as measured in kilometres. Suppose city A is 500 km. distant from your present location and city B is 1000 km. distant, so B is twice as far as A. Similarly if C is 200 km. distant and D is 400 km. distant, D is twice as far away as C. These ratios of 1000/500 and 400/200 are each equal to 2, and this value of 2 represents the same relative ratio between the two pairs of cities.

Interval scales are generally ratio scales, so long as the zero point of the scale is not arbitrary. In the above example, a distance of 0 kilometres means no distance at all, so the zero point of any distance measurement is not arbitrary. Zero distance represents an absence of any distance. The same is true with measures in dollars. An income or amount of money of $0 represents no money at all. Since measurements in dollars have an interval level, and since the zero point is not arbitrary, measurements in dollars have a ratio scale of measurement.

Temperature is a prime example of an interval scale that does not have ratio scale of measurement. In the case of temperature on the Celsius or Fahrenheit scale, the zero point is arbitrary and does not represent the absence of temperature. As a result, ratios of temperature in degrees Celsius are not necessarily meaningful in a ratio sense. Does it make sense to say that it is twice as hot at 40° C. as it is at 20° C.? It is much hotter at 40° C., but it is not clear what it means to say that it is exactly twice as hot as at 20° C. In the case of comparing positive and negative temperatures, ratios make little sense and are not meaningful.

In the case of scales such as IQ or attitude scales, while they might be treated as interval scales for purpose of statistical analysis, it would be a more serious error to treat these as ratio scales. For example, what would it mean to say that someone with an IQ of 140 was twice as intelligent as someone with an IQ of 70? Similarly with attitude, if there is five point scale from 1, indicating strongly disagree, to 5, indicating strongly agree, does it make sense to say that a response of 4 is twice that of 2? In this case, 2 is on the disagree side and 4 is on the agree side, so a ratio of two such values has little meaning.

When constructing scales, it may be possible to construct the scale so the intervals between values are meaningful (interval level) and the zero point really means none of the characteristic in question (ratio level). If this is possible, then the scale is a ratio scale, the highest level of measurement. In this case, it is possible to add, subtract, multiply, and divide values of the variable. All the statistical procedures can be legitimately used for a variable or variables with a ratio scale of measurement.

Conclusion to scales of measurement

When working with quantitative data, it is important to be aware of the type of scale associated with each variable. Statistical procedures and interpretations differ depending on the type of scale associated with a variable. The key issues associated with different levels of measurement are as follows.

- **Presentation**. Quantitative data is presented differently for different types of measurement. In the case of a scale that is no more than nominal, tables usually list the names or codes associated with the different categories of the variable. The same is often the case for variables measured at the ordinal level. In the case of interval and ratio level scales, it is likely that the presentation of variables will be organized into numerical categories. For example, a table of income distribution might be organized into numerical intervals such as $0 to $20,000, $20,000 to $40,000, and $40,000 and over.

- **Summary statistics**. The type of summary statistical measure that can be meaningfully calculated and presented differs by type of scale. For a variable with not more than a nominal scale, the only summary measure that is meaningful is the mode, the value that occurs most frequently. In the case of an ordinal scale, the median (middle value) can be computed and is meaningful. For interval and ratio scales, the average value, or mean, can be calculated; other summary measures, such as the standard deviation, are also useful. We will define and discuss these in Modules 3 and 4 and Chapter 5 of the text.

- **Statistical techniques**. Different statistical procedures are appropriate for variables with different scales of measurement. In the case of nominal scales, data can be organized into tables, but there are only limited forms of statistical analysis that can be conducted with these tables. For ordinal, interval, and ratio scales, it is possible to use more complex statistical techniques that yield greater insight into the structure of the data and the connection among variables. Different techniques will be examined later in the semester, in Modules 6-8 and Chapters 8-10 of the text.

The reason why scales differ in forms of measurement is partly a result of the nature of the phenomenon or characteristic being investigated and partly the way that the researcher asks questions and constructs variables. Where possible, it is advisable for the researcher to construct variables at the highest level possible, that is, construct variables at the interval or ratio level of measurement. In the case of measures of attitudes or opinions, or some basic human characteristics such as sex and region of origin, this cannot be done. But in other cases, such as income in dollars or education in years, it is advisable to measure phenomena at the interval and ratio level of measurement.

Discrete and continuous

See text, section 3.3, pp. 72-75.

Another way of examining characteristics of variables is to distinguish variables that are discrete from those that are continuous. The former are variables with only a discrete, or countable, set of possible values. The latter are variables that are continuous, with an infinite and uncountable set of possible values. This distinction is not as key to selecting different forms of statistical analysis as is the distinction between the nominal, ordinal, interval, and ratio levels of measurement. For some purposes though, the distinction between discrete or continuous has relevance when presenting and analyzing data.

This section of these notes defines discrete and continuous and gives examples of each type of variable.

Discrete variable

A variable that is discrete has values that are discrete or different, in that the different values can be clearly distinguished from each other. A formal definition is as follows.

> **Definition of discrete scale**. A discrete variable is a variable that can take on only a countable number of values.

If is is possible to count all the possible values of a variable, it is considered discrete.

Many variables are discrete since they have only a few possible values. For example sex has only two values, male and female. These are distinct and can be counted. Political party supported is discrete since it can take on only values such as Liberal, NDP, Saskatchewan, Green, Conservative, Natural Law, and so on. There could be many such parties, but each political party is a separate and distinct entity, so all possible parties can be counted. For undergraduate students at the University of Regina, the Faculty in which they are registered is a discrete variable. Students must be registered somewhere – Arts, Fine Arts, Engineering, etc. There may be a catch-all category such as "Other" for those with unusual registration, but the number of possibile types of registration can be counted. In the case of a variable such as religion or ethnicity, there may be a very large number of possible values, but all could potentially be counted, if a researcher had sufficient time and resources. Any variable with a number of distinct and countable characteristics is a discrete variable.

It may be possible for a discrete variable to have an extremely large number of values, even an infinite number, but to be discrete they must be countable. That is, each possible value must be a distinct and independent entity for the variable to have a discrete set of values. Any measurement of number of people, for example, the number of students in a class, is a discrete variable – each student is a distinct human being, and the number of students can be counted. The number of stars in the universe may be infinite, but each star is a distinct and independent entity, distinguishable from other stars. As a result, the number of stars can be considered a discrete variable. In most cases where the term "number of" is employed, this means that the variable is discrete.

The integers of our number system, 1, 2, 3, 4, 5, ... are countable, and may be used to define what counting means. The set of all possible integers is infinite in that one can always find an integer one larger than any given integer. Any variable whose possible categories can be matched with the integers has a countable set of categories. As a result, any variable whose possible values can be matched with the integers is a discrete variable.

Continuous variable

In contrast to discrete variables, if the number of possible values of a variable cannot be counted, then the variable is continuous. For example, the number of possible temperatures or heights cannot be counted – these are characteristics that are inherently continuous. A formal definition is as follows.

> **Definition of continuous variable**. A continuous variable is a variable that can assume any value along some line interval.

Any characteristic that can be matched up with all the points along a line can be considered continuous. The number of points on a line cannot be counted, rather the variable can move continuously along the line, to any point on the line. In the case of age, while we ordinarily round our age to age as of last birthday, or to nearest birthday, an individual ages in a continuous manner, as time passes in a continuous manner, not in discrete jumps. The rounding of age to nearest year is merely a convenience that makes it easier to report age. So long as age represents a period of time, age is a continuous variable.

Liquids are continuous in nature, in that they are not divided into discrete parts, but flow continuously. Measures of volume, such as litres or gallons, are thus continuous measures.

It is common for statistical analysts to round values of continuous variables to the nearest integer or nearest ten, hundred, or thousand. This may mean the variable is organized and presented in a countable set of categories. For example, while children grow in height in a continuous fashion until they reach their adult height, the height may be reported by rounded to the nearest centimetre or inch. Just because the variables have been rounded does not mean they are discrete. Situations such as this can be described as variable being continuous, but reported in a discrete set of categories or intervals.

Two commonly used social science variables that may be confusing in this respect are attitudes and income. Each of these variables is inherently continuous, but researcher often report these in only a discrete fashion. Consider the following line, representing the set of possible attitudes to a statement about a social or political issue.

Strongly disagree Strongly agree

Suppose a researcher asks an individual to state his or her position about the statement, by requesting the that the individual to select a point on the line that he or she considers to correspond to his or her view. This might be a useful way of obtaining data on attitudes or opinions. Since all the possible attitudes or opinions can be matched with points along a continuous line, this demonstrates that attitudes are continuous variables. But researchers usually obtain and report only a discrete set of response categories such as agree, neutral, or disagree.

In the case of income, the situation is similar. Incomes are ordinarily measured in dollars and the dollar value can be anywhere from zero (very poor) to many millions of dollars (very rich). While the smallest monetary unit in circulation is one cent, there is no reason why a monetary value cannot be calculated in fractions of a cent. If you examine the business pages of the newspaper, you will note that foreign exchange values are often given to several decimal places. As a result, income or any other variable measured in monetary terms (dollars), can be considered continuous in nature. However, we usually round these values and report incomes to the nearest dollar.

Conclusion to types of variables

When you encounter a new variable in a data set, one that you have not used before, one of the first questions to ask is how it was measured, what type of scale it has (nominal, ordinal, interval, or ratio) and whether it is discrete or continuous. Conventions about proper ways of presenting data and conducting the proper forms of statistical analysis may differ depending on the type of measurement and whether the variable is discrete or continuous. If you are constructing variables yourself, it is important to carefully consider what form of measurement you will use for constructing the variables.

The distinction between discrete and continuous is less important for later parts of this course than is the scale of measurement. However, the distinction between discrete and continuous does affect some aspects of data presentation an statistical analysis. These situations will be discussed at relevant points of the notes.

The following sections of Module 2 examine conventions, guidelines, and rules for presenting variables. As you encounter new variables in the rest of Module 2 always take note of the type of variable. This will help you determine how information about the values of the variables are most appropriately presented.