Social Studies 201
Winter 2005

Answers to Problem Set 5

1. **Interval estimates**

   (a) Let μ be the true mean age at first marriage for all Saskatchewan males in 2001. From Table 1, the sample size is $n = 321$ and the test statistic is $\bar{X}$. Given this large sample size, $\bar{X}$ has a normal distribution with mean μ and standard deviation $\sigma/\sqrt{n}$, where the sample standard deviation $s = 8.07$ is used as an estimate of σ. For 85% confidence, the Z value is 1.44 and the 85% interval estimate is:

$$\begin{aligned}
\bar{X} \pm Z\frac{\sigma}{\sqrt{n}} &= \bar{X} \pm 1.44\frac{8.07}{\sqrt{321}} \\
&= \bar{X} \pm 1.44\frac{8.07}{17.916} \\
&= \bar{X} \pm (1.44 \times 0.450) \\
&= \hat{X} \pm 0.649 \\
&= 27.52 \pm 0.65
\end{aligned}$$

   or from 26.87 to 28.17.

   For females, the method is the same. Again the sample size $n = 352$ is large, so the sample mean has a normal distribution. Also, since n is large, s is used as an estimate of σ, and the 85% interval estimate is

$$\begin{aligned}
\bar{X} \pm Z\frac{\sigma}{\sqrt{n}} &= \bar{X} \pm 1.44\frac{7.68}{\sqrt{352}} \\
&= \bar{X} \pm 1.44\frac{7.68}{18.762} \\
&= \bar{X} \pm (1.44 \times 0.409) \\
&= \hat{X} \pm 0.589 \\
&= 24.67 \pm 0.59
\end{aligned}$$

or from 24.08 to 25.26.

For males, the 85% interval estimate is that the true mean age at first marriage is between 26.9 and 28.2. The actual mean age at first marriage in 1976, 27.9 years, is within this interval. As a result, for males there is insufficient evidence to conclude that the mean age of first marriage for Saskatchewan males in 2001 differs from the Canadian mean for males in 1976. For the females, the mean age at first marriage in 1976 was 25.3 years. This is just at the edge of the interval estimate here, from 24.1 to 25.3 years. Again, this is consistent with a claim that the mean age at first marriage for Saskatchewan females in 2001 does not differ from the Canadian mean in 1976.

Question 3.a. contains a more direct test of the above conclusions.

(b) For the first part of this question, let μ be the true mean hours of sleep for males, and then in the second part, μ represents the mean hours of sleep for females. From Table 2, for both males and females the sample size is small, $n = 12$ for males and $n = 25$ for females. Both of these sample sizes are less than $n = 30$ so that, assuming the distribution of hours of sleep is normal, each of the sample means, $\bar{X}$ has a t-distribution with mean μ, standard deviation $s/\sqrt{n}$, and $n-1$ degrees of freedom. For males, with a sample size of $n = 12$, or $n - 1 = 12 - 1 = 11$ degrees of freedom, $t = 1.796$. For females, the sample size is $n = 25$, so there are $n-1 = 25-1 = 24$ degrees of freedom, and $t = 1.711$. Both these t-values are for 90% confidence.

For males, the 90% interval estimate is

$$\begin{aligned}
\bar{X} \pm t\frac{\sigma}{\sqrt{n}} &= \bar{X} \pm 1.796\frac{1.14}{\sqrt{12}} \\
&= \bar{X} \pm 1.796\frac{1.14}{3.464} \\
&= \bar{X} \pm (1.796 \times 0.329) \\
&= \hat{X} \pm 0.591 \\
&= 6.25 \pm 0.59
\end{aligned}$$

or from 5.66 to 6.84.

For females, the 90% interval estimate is

$$
\begin{aligned}
\bar{X} \pm t\frac{\sigma}{\sqrt{n}} &= \bar{X} \pm 1.711\frac{1.34}{\sqrt{25}} \\
&= \bar{X} \pm 1.711\frac{1.34}{5.000} \\
&= \bar{X} \pm (1.711 \times 0.268) \\
&= \hat{X} \pm 0.462 \\
&= 6.84 \pm 0.46
\end{aligned}
$$

or from 6.38 to 7.30.

From these two intervals, the true mean for males could be anywhere between 5.7 and 6.8 hours of sleep daily and for females the true mean could be anywhere between 6.4 and 7.3 hours of sleep daily. From Table 2, while males appear to sleep less than females, the difference is not large, the sample sizes are small, and the intervals overlap, so there is not strong evidence that males and females sleep different numbers of hours.

(c) Let p be the true proportion of Quebec residents who are satisfied. The sample in Table 3 indicates that 22% + 54% = 76% or $\hat{p} = 0.76$ of those in the sample are satisfied. Since $n = 498$ is large ($n = 498 > 5/0.24 = 20.8$), the sample proportion $\hat{p}$ is normally distributed with mean p and standard deviation $\sqrt{pq/n}$. For the 94% interval estimate, $Z = \pm 1.88$. By using $p = q = 0.5$, the widest possible interval for any of the proportions is obtained. Alternatively, $\hat{p} = 0.76$ and $\hat{q} = 1 - \hat{p} = 1 - 0.76 = 0.24$ could be used for p and q in the estimate of the standard deviation of $\hat{p}$. The interval estimate is:

$$
\begin{aligned}
\hat{p} \pm Z\sqrt{\frac{pq}{n}} &= \hat{p} \pm 1.88\sqrt{\frac{0.5 \times 0.5}{498}} \\
&= \hat{p} \pm 1.88\sqrt{0.000502} \\
&= \hat{p} \pm (1.88 \times 0.02241) \\
&= \hat{p} \pm 0.0421 \\
&= 0.76 \pm 0.04
\end{aligned}
$$

or from 0.72 to 0.80. The 90% interval estimate for the percentage of Quebec respondents who are satisfied is from 72% to 80%.

If $\hat{p} = 0.76$ and $\hat{q} = 0.24$ had been used instead of $p = q = 0.5$ in $\sqrt{pq/n}$, the interval would be a little narrower. The interval would be ± 0.036 around $\hat{p}$, but rounded this would be ± 0.04, so the interval would not be much different.

For the rest of Canada, the method is the same, and the sample size is even larger, $n = 1,529$, so again the sample proportions have a normal distribution. The percentage who are satisfied is $16\% + 65\% = 81\%$, or a proportion of 0.81. Using $p = q = 0.5$ in the estimate of the standard error of $\hat{p}$, for the rest of Canada the interval estimate is:

$$\begin{aligned} \hat{p} \pm Z\sqrt{\frac{pq}{n}} &= \hat{p} \pm 1.88\sqrt{\frac{0.5 \times 0.5}{1,529}} \\ &= \hat{p} \pm 1.88\sqrt{0.000164} \\ &= \hat{p} \pm (1.88 \times 0.01279) \\ &= \hat{p} \pm 0.0240 \\ &= 0.81 \pm 0.02 \end{aligned}$$

or from 0.79 to 0.83. The 90% interval estimate for the percentage of those in the rest of Canada who are satisfied is from 79% to 81%. Using $\hat{p} = 0.81$ and $\hat{q} = 0.19$ in the estimate of the standard error, the interval would be little different.

From these two interval estimates, there is not strong evidence that the two regions differ in the percentage of support. While those surveyed in Quebec express less support, the two intervals overlap, so the true proportion of supporters for each region could be the same. Having said this, among those surveyed, support was lower in Quebec than in the rest of Canada, but the difference in percentages of support was not large.

2. **Sample sizes**

(a) This is the sample size required to estimate a mean. The required sample size is

$$n = \left(\frac{Z\sigma}{E}\right)^2$$

where E is the accuracy required. In order to be correct eighteen in twenty times, or 90% of the time, the Z-value is 1.645. Using the larger of the two standard deviations in Table 2, the estimate of σ is 1.34.

If the accuracy of the estimate is to be one-quarter hour, or $E = 0.25$, then the required sample size is

$$\begin{aligned} n &= \left(\frac{Z\sigma}{E}\right)^2 && (1)\\ &= \left(\frac{1.645 \times 1.34}{0.25}\right)^2 && (2)\\ &= 8.8172^2 && (3)\\ &= 77.7 && (4)\end{aligned}$$

or $n = 78$.

For an estimate to be accurate to within five minutes, $E = 5/60 = 1/12 = 0.0833$ hours. The required sample size is

$$\begin{aligned} n &= \left(\frac{Z\sigma}{E}\right)^2 && (5)\\ &= \left(\frac{1.645 \times 1.34}{0.0833}\right)^2 && (6)\\ &= 26.4516^2 && (7)\\ &= 699.687 && (8)\end{aligned}$$

or $n = 700$.

(b) For this question, either the formula for the sample size for a proportion or the interval estimate for a proportion can be used. For verifying the statement, I use the formula for sample size. The for determining the margin of error for Quebec and the rest of Canada, I use the interval estimate method.

For estimating a proportion, the formula for sample size is

$$n = \left(\frac{Z}{E}\right)^2 pq$$

where E is the accuracy of the estimate. The claim is that the accuracy is 2.2 percentage points or $E = 0.022$. 19 in 20 times is equivalent to $19/20 = 0.95$ or 95%, so the Z-value for this confidence level is 1.96. Using $p = q = 0.5$ gives the largest possible sample size for any given accuracy and confidence level. Using these values in the formula gives

$$\begin{aligned} n &= \left(\frac{Z}{E}\right)^2 pq && (9) \\ &= \left(\frac{1.96}{0.022}\right)^2 0.5 \times 0.5 && (10) \\ &= 89.091^2 \times 0.25 && (11) \\ &= 7937.19 \times 0.25 && (12) \\ &= 1,984.3 && (13) \end{aligned}$$

or $n = 1,985$. While this sample size differs a little from the stated sample size of $n = 2,027$, it is very similar, so this appears to be a more or less correct claim.

For the margins of error, the interval estimates are provided. These are as follows.

$$\begin{aligned} \hat{p} \pm Z\sqrt{\frac{pq}{n}} &= \hat{p} \pm 1.96\sqrt{\frac{0.5 \times 0.5}{498}} \\ &= \hat{p} \pm 1.96\sqrt{0.000502} \\ &= \hat{p} \pm (1.96 \times 0.02241) \\ &= \hat{p} \pm 0.044 \end{aligned}$$

$$\begin{aligned} \hat{p} \pm Z\sqrt{\frac{pq}{n}} &= \hat{p} \pm 1.96\sqrt{\frac{0.5 \times 0.5}{1,529}} \\ &= \hat{p} \pm 1.96\sqrt{0.0001635} \\ &= \hat{p} \pm (1.96 \times 0.012787) \\ &= \hat{p} \pm 0.025 \end{aligned}$$

The margin of error for Quebec is ±4.4%, nineteen in twenty times. For the rest of Canada, the comparable margin is plus or minus 2.5 percentage points, nineteen in twenty times.

3. **Hypothesis tests**

(a) Let μ be the true mean age at first marriage for Saskatchewan males in 2001. For males, the hypotheses are:

$$H_0 \; : \; \mu = 27.9 \quad \text{(same as 1976 Canadian mean)}$$

$$H_1 \; : \; \mu \neq 27.9 \quad \text{(differs from 1976 Canadian mean)}$$

This is a two-tailed test with the null hypothesis being that there is no change in the mean age at first marriage for males and the alternative hypothesis being that the mean age in 2001 differs from that in 1976. The test statistic is the sample mean, $\bar{X}$. Since $n = 321$ is large (over 30), the sample mean has a normal distribution with mean μ and standard deviation $\sigma/\sqrt{n}$, by the Central Limit Theorem. The significance level is $\alpha = 0.10$ and since the test is a two-tailed test the critical Z-value is ±1.645. H_0 is rejected for all $Z < -1.645$ or $Z > +1.645$. From the data in Table 1, the Z-value is

$$\begin{aligned} Z &= \frac{\bar{X} - \mu}{s/\sqrt{n}} \\ &= \frac{27.52 - 27.9}{8.07/\sqrt{321}} \\ &= \frac{-0.38}{8.07/17.916} \\ &= \frac{-0.38}{0.450} \\ &= -0.844 \end{aligned}$$

Since this Z-value is greater than -1.645 but less than +1.645, there is not sufficient evidence to reject the null hypothesis at the 0.10 level of significance. The conclusion is that, at the 0.10 level of significance, there is not evidence to indicate that the mean age at first marriage for Saskatchewan males in 2001 is any different from the mean age for Canada in 1976.

For females, the method is the same and the only difference is that the question asks whether the mean age at first marriage for females is lower than the 1976 Canadian mean for females. This changes the alternative hypothesis into a one-directional test as follows.

$$H_0 : \mu = 25.3 \quad \text{(same as 1976 Canadian mean)}$$
$$H_1 : \mu < 25.3 \quad \text{(less than 1976 Canadian mean)}$$

This is a one-tailed test with the null hypothesis being that there is no change in the mean age at first marriage for females and the alternative hypothesis is that the mean age in 2001 is lower than the Canadian mean in 1976. The test statistic is the sample mean, $\bar{X}$. Since $n = 352$ is large (over 30), the sample mean has a normal distribution with mean μ and standard deviation $\sigma/\sqrt{n}$, by the Central Limit Theorem. The significance level is $\alpha = 0.10$ and since the test is a one-tailed test the critical Z-value is -1.28. H_0 is rejected for all $Z < -1.28$. From the data in Table 1, the Z-value is

$$\begin{aligned} Z &= \frac{\bar{X} - \mu}{s/\sqrt{n}} \\ &= \frac{24.67 - 25.3}{7.68/\sqrt{352}} \\ &= \frac{-0.63}{7.68/18.762} \\ &= \frac{-0.63}{0.409} \\ &= -1.539 < -1.28 \end{aligned}$$

The Z-value is to the left of -1.28 and is in the critical region. As a result, the hypothesis that there is no difference in the mean age at first marriage can be rejected and the hypothesis that the mean age at first marriage for Saskatchewan females in 2001 is less than the Canadian mean in 1976 can be supported. This conclusion is made at the 0.10 level of significance.

(b) For this question, the sample sizes are $n = 12$ for males and $n = 25$ for females. Both these are small sample sizes so the t-distribution is used for these hypothesis tests. In order to use the t-distribution, it is necessary to assume that the populations are normally distributed. In this case, the assumption is that hours of sleep are normally distributed – this may not be exactly true, but may be close to being a reasonable description of the distribution of hours of sleep. In terms of the hypotheses, for both males and females the question is whether the true mean is less than seven hours, so each test is a one-tailed test in the negative, or less than, direction.

Let μ be the true mean hours of sleep for all Saskatchewan males aged 20-24 with one or more children in the household. The hypotheses are

$$\begin{aligned} H_0 &: \mu = 7 \\ H_1 &: \mu < 7 \end{aligned}$$

The test statistic is the sample mean, $\bar{X}$, and as noted above this statistic has a t-distribution. Since $n = 12$, there are $n - 1 = 12 - 1 = 11$ degrees of freedom. Given the 0.05 level of significance and a one tailed test in the negative direction, the critical region is all t-values less than -1.796.

From the data in Table 2, the t-value for the male sample mean is

$$\begin{aligned} t &= \frac{\bar{X} - \mu}{s/\sqrt{n}} \\ &= \frac{6.25 - 7}{1.14/\sqrt{12}} \\ &= \frac{-0.75}{1.14/3.464} \\ &= \frac{-0.75}{0.329} \\ &= -2.279 < -1.796 \end{aligned}$$

This t-value is less than -1.796, so the the null hypothesis is rejected at the 0.05 level of significance. At the 0.05 level of significance the sample provides evidence that the mean hours of sleep for all Saskatchewan males aged 20-24 with one or more children in the household is less than seven hours.

For females the method is identical, with the only difference being the values of the statistics from the sample. The sample size $n = 25$ indicates $n-1 = 25-1 = 24$ degrees of freedom and, with a significance level of 0.05, the critical region is all t-values to the left of $t = -1.711$. From the data in Table 2,

$$\begin{aligned} t &= \frac{\bar{X} - \mu}{s/\sqrt{n}} \\ &= \frac{6.84 - 7}{1.34/\sqrt{25}} \\ &= \frac{-0.16}{1.34/5.000} \\ &= \frac{-0.16}{0.268} \\ &= -0.597 > -1.711 \end{aligned}$$

and this t-value is to the right of -1.711 so is not in the critical region. The conclusion is thus to not reject the null hypothesis that the mean hours of sleep for Saskatchewan females aged 20-24 with one or more children in the household is seven hours. This conclusion is made at the 0.05 level of significance.

For males, the sample mean was somewhat less than that for the females. Even though the sample size for males was smaller than that for females, the difference from the hypotesized mean of seven hours was enough greater for males than for females, to reject the null hypothesis. In the case of females, the sample mean was very little different from seven hours, so the data from the sample do not provide sufficient evidence to conclude that Saskatchewan females sleep less than seven hours, on average.

(c) Let p be the true proportion of Quebec residents who were satisfied with the health agreement. The question is to test whether this proportion differs from 0.80, or 80%. Given that the question is whether there is a difference from 0.80, this is a two-tailed or two-directional test. The hypotheses are

$$H_0 \;:\; p = 0.80$$

$$H_1 \;:\; p \neq 0.80$$

The sample size is $n = 498$ for Quebec and the test statistic is $\hat{p}$, where $\hat{p} = 0.76$ (sum of very and somewhat satisfied), $\hat{q} = 1 - \hat{p} = 1 - 0.76 = 0.24$. To ensure this is a large sample size, n divided by the smaller of p or q gives $5/0.24 = 20.8$ and $n = 498 > 20.8$ so this is a large sample size and $\hat{p}$ has a normal distribution with mean p and standard deviation $\sqrt{pq/n}$. At $\alpha = 0.10$ significance, with a two-tailed test, the critical $Z = 1.645$. When the sample $Z < -1.645$ or $Z > +1.645$, the null hypothesis is rejected but when $-1.645 < Z < +1.645$, the null hypothesis is not rejected. From the sample data,

$$\begin{aligned} Z &= \frac{\hat{p} - p}{\sqrt{pq/n}} \\ &= \frac{0.76 - 0.80}{\sqrt{(0.80 \times 0.20)/498}} \\ &= \frac{-0.04}{0.01792} \\ &= -2.232 < -1.645 \end{aligned}$$

Since the Z-value is less than -1.645, the conclusion is to reject the null hypothesis at the 0.10 level of significance. At the 0.10 level of significance the data provide evidence that the proportion of Quebec residents who support the health agreement is less than 80%.

For the rest of Canada, the method is the same except that the alternative hypothesis is a one-directional test in the positive direction. Let p be the true proportion of residents in the rest of Canada who were satisfied with the health agreement. The question is to test whether this proportion exceeds 0.80, or 80%. The hypotheses are

$$H_0 \;:\; p = 0.80$$
$$H_1 \;:\; p > 0.80$$

The sample size is $n = 1,598$ so this is a large sample size and $\hat{p}$ has a normal distribution with mean p and standard deviation $\sqrt{pq/n}$. At $\alpha = 0.10$ significance, with a one-tailed test, the critical $Z = 1.28$. When the sample $Z > +1.28$, the null hypothesis is rejected but when $Z < +1.28$, the null hypothesis is not rejected. From the sample data,

$$\begin{aligned}
Z &= \frac{\hat{p} - p}{\sqrt{pq/n}} \\
&= \frac{0.81 - 0.80}{\sqrt{(0.80 \times 0.20)/1,529}} \\
&= \frac{0.01}{0.01023} \\
&= 0.978 < 1.28
\end{aligned}$$

The Z-value is less than the critical Z-value of 1.28, so the conclusion is not to reject the null hypothesis. At the 0.10 significance level, there is insufficient evidence from the sample to state that the proportion of rest of Canada residents who support the health agreement exceeds 80%.

4. Computer problem for Problem Set 4

1. Comments included after each table, with summary at the end.

Part i.

Case Processing Summary

	Cases					
	Valid		Missing		Total	
	N	Percent	N	Percent	N	Percent
Lower Tuition	676	95.6%	31	4.4%	707	100.0%

Descriptives

			Statistic	Std. Error
Lower Tuition	Mean		4.09	.048
	95% Confidence Interval for Mean	Lower Bound	3.99	
		Upper Bound	4.18	
	5% Trimmed Mean		4.21	
	Median		5.00	
	Variance		1.560	
	Std. Deviation		1.249	
	Minimum		1	
	Maximum		5	
	Range		4	
	Interquartile Range		1.00	
	Skewness		-1.270	.094
	Kurtosis		.479	.188

For the variable UED4, the 95% interval estimate for the true mean opinion concerning lowering tuition is from 3.99 to 4.18. If 4 represents a response of somewhat agreeing, these data suggest that undergraduates express reasonably strong agreement with this statement.

Part

ii.

SEX OF RESPONDENT

Case Processing Summary

		Cases					
		Valid		Missing		Total	
	SEX OF RESPONDENT	N	Percent	N	Percent	N	Percent
Lower Tuition	MALE	253	96.6%	9	3.4%	262	100.0%
	FEMALE	423	95.1%	22	4.9%	445	100.0%

Descriptives

	SEX OF RESPONDENT			Statistic	Std. Error
Lower Tuition	MALE	Mean		3.92	.081
		97% Confidence Interval for Mean	Lower Bound	3.75	
			Upper Bound	4.10	
		5% Trimmed Mean		4.03	
		Median		4.00	
		Variance		1.673	
		Std. Deviation		1.293	
		Minimum		1	
		Maximum		5	
		Range		4	
		Interquartile Range		2.00	
		Skewness		-1.024	.153
		Kurtosis		-.123	.305
	FEMALE	Mean		4.18	.059
		97% Confidence Interval for Mean	Lower Bound	4.05	
			Upper Bound	4.31	
		5% Trimmed Mean		4.31	
		Median		5.00	
		Variance		1.472	
		Std. Deviation		1.213	
		Minimum		1	
		Maximum		5	
		Range		4	
		Interquartile Range		1.00	
		Skewness		-1.449	.119
		Kurtosis		1.022	.237

For males, the mean response is 3.92, and the 97% interval estimate runs from 3.75 to 4.10. For females, the mean response is 4.18, with the 97% interval estimate from 4.05 to 4.31. While it is possible that males and females have the same view on the issue of reducing tuition, the intervals overlap only a little. The true mean response for both male and female undergraduates could be in this overlap region, but this is a fairly small overlap. As a result, there is reasonable assurance that females are more in agreement with reducing tuition than are males.

Part iii.

Case Processing Summary

	federal political preference	Cases					
		Valid		Missing		Total	
		N	Percent	N	Percent	N	Percent
Lower Tuition	Liberal	185	96.9%	6	3.1%	191	100.0%
	NDP	90	96.8%	3	3.2%	93	100.0%
	Conservative	87	98.9%	1	1.1%	88	100.0%
	None	147	95.5%	7	4.5%	154	100.0%

The means and 90% intervals are given in the table on the following page and summarized here. I have reordered them in order of greatest support for reducing tuition (Liberal) to least support for reducing tuition (Conservative).

Liberal	**4.24**	**(4.10, 4.38)**
None	**4.15**	**(3.98, 4.32)**
NDP	**4.09**	**(3.87, 4.30)**
Conservative	**3.79**	**(3.56, 4.03)**

Given that there are four groups, some overlap of the intervals is to be expected. What these results show is that those who support the Conservative party are least in support of reducing tuition. The interval for the Conservatives does not overlap that of the Liberals, so there is reasonably strong evidence that Conservative supporters as a whole really do express less support for reducing tuition that do Liberals. The other groups are in between these extremes and there is not all that great a difference between those who support the NDP and those who support no political party. These intervals are fairly similar, so it would be difficult to say there is any large difference of views between these two groups.

In summary, on average students support a reduction in tuition, expressing fairly strongly agreement, with the mean just above 4 on the five-point scale. Females express stronger support than do males and, in terms of political preference, Liberals are most in support, Conservatives least, with NDP and supporters of no party in between. Many of the intervals overlap, so the differences among these groups are not large, so that definitive statements cannot be made about differences in group support.

Descriptives

	federal political			Statistic	Std. Error
Lower Tuition	Liberal	Mean		4.24	.085
		90% Confidence Interval for Mean	Lower Bound	4.10	
			Upper Bound	4.38	
		5% Trimmed Mean		4.38	
		Median		5.00	
		Variance		1.345	
		Std. Deviation		1.160	
		Minimum		1	
		Maximum		5	
		Range		4	
		Interquartile Range		1.00	
		Skewness		-1.617	.179
		Kurtosis		1.673	.355
	NDP	Mean		4.09	.130
		90% Confidence Interval for Mean	Lower Bound	3.87	
			Upper Bound	4.30	
		5% Trimmed Mean		4.21	
		Median		5.00	
		Variance		1.520	
		Std. Deviation		1.233	
		Minimum		1	
		Maximum		5	
		Range		4	
		Interquartile Range		1.25	
		Skewness		-1.276	.254
		Kurtosis		.577	.503
	Conservative	Mean		3.79	.141
		90% Confidence Interval for Mean	Lower Bound	3.56	
			Upper Bound	4.03	
		5% Trimmed Mean		3.88	
		Median		4.00	
		Variance		1.724	
		Std. Deviation		1.313	
		Minimum		1	
		Maximum		5	
		Range		4	
		Interquartile Range		2.00	
		Skewness		-.742	.258
		Kurtosis		-.705	.511
	None	Mean		4.15	.102
		90% Confidence Interval for Mean	Lower Bound	3.98	
			Upper Bound	4.32	
		5% Trimmed Mean		4.28	
		Median		5.00	
		Variance		1.539	
		Std. Deviation		1.241	
		Minimum		1	
		Maximum		5	
		Range		4	
		Interquartile Range		1.00	
		Skewness		-1.358	.200
		Kurtosis		.701	.397

NOTE: For all the tests in questions 2 and 3, the sample size and degrees of freedom are large (well over 30), so the t-value listed on the printout is really a Z-value. I will refer to it throughout as a Z-value.

2. The tests are as follows.

i. Hourly pay

T-Test

One-Sample Statistics

	N	Mean	Std. Deviation	Std. Error Mean
Hourly pay in dollars	384	9.7200	5.42857	.27703

One-Sample Test

	Test Value = 10					
					95% Confidence Interval of the Difference	
	t	df	Sig. (2-tailed)	Mean Difference	Lower	Upper
Hourly pay in dollars	-1.011	383	.313	-.2800	-.8247	.2647

For this question, the true mean, μ, is the mean pay for all undergraduates with jobs. The hypotheses are

H0: $\mu = 10$ Mean wages equal \$10 per hour
H1: $\mu \neq 10$ Mean wages differ from \$10 per hour

Picking a significance level of 0.10, the critical region is all Z-values greater than +1.645 or less than -1.645. (For 0.05 significance, the critical values are plus or minus 1.96). The Z-value from this test is Z = -1.011, and this is not in the critical region. As a result, the null hypothesis is not rejected. At the 0.10 level of significance, there is not strong evidence that the hourly pay of undergraduates with jobs differs from \$10 per hour. The pay for those in the sample is \$9.72 per hour, and while this is less than \$10 per hour, the test shows that it is not enough different from \$10 per hour to conclude that pay for all undergraduates differs from \$10 per hour. This conclusion is made at the 0.10 level of significance.

ii. Debt

One-Sample Statistics

	N	Mean	Std. Deviation	Std. Error Mean
Debt before Fall 1998	606	3134.95	6635.048	269.530
Debt after Winter 1999	590	4612.88	8155.726	335.766

One-Sample Test

	Test Value = 4000					
					95% Confidence Interval of the Difference	
	t	df	Sig. (2-tailed)	Mean Difference	Lower	Upper
Debt before Fall 1998	-3.209	605	.001	-865.05	-1394.38	-335.72
Debt after Winter 1999	1.825	589	.068	612.88	-46.56	1272.33

The two tests are shown together here. μ is the true mean debt for each of the two time periods.

In the case of Fall 1998, the hypotheses are

H0: $\mu = \$4,000$ Mean debt was \$4,000 at start of Fall 1998
H1: $\mu < \$4,000$ Mean debt was less than \$4,000 at start of Fall

If the significance level is 0.01, the critical region is $Z<-2.33$. In this case, $Z = -3.209 < -2.33$, so reject the null hypothesis and conclude that mean debt was less than \$4,000. The mean debt of \$3,135 is enough less than the hypothesized mean debt of \$4,000 to conclude that the overall mean debt for all undergraduates was less than \$4,000 at the start of the Fall 1998 semester (0.01 significance).

In the case of Winter 1998, the hypotheses are

H0: $\mu = \$4,000$ Mean debt will be \$4,000 at end of Winter 1998
H1: $\mu > \$4,000$ Mean debt will be greater than \$4,000 at end of Winter 1998

For this test, the sample mean is \$4,613, and the associated Z-value is 1.825, At the 0.05 level of significance, the critical Z-value is 1.645. At 0.05 significance, the hypothesis that the mean debt will be greater than \$4,000 can be supported. Note though that if the 0.01 level of significance is used, so the critical Z is 2.33, the null hypothesis is not rejected. As a result, there is reasonably strong evidence that the mean debt will be greater than \$4,000, but the evidence is not strong enough to reject the \$4,000 hypothesis at 0.01 significance.

3. Weekly study hours. The four tests are as follows. In each case, μ is the true mean study hours for all undergraduate University of Regina students. The null hypothesis in each case is that μ equals the value specified, for example

H0: μ = 15 for i., μ = 16 for ii, and so on.

The alternative hypothesis for part i is

H1: μ > 15

For parts ii and iii, the alternative hypothesis is that μ ≠ 16 and μ ≠ 17; finally, for the last part the alternative hypothesis is that μ < 18.

T-Test

One-Sample Statistics

	N	Mean	Std. Deviation	Std. Error Mean
Study Hours	668	16.61	11.849	.458

One-Sample Test

	Test Value = 15					
					95% Confidence Interval of the Difference	
	t	df	Sig. (2-tailed)	Mean Difference	Lower	Upper
Study Hours	3.502	667	.000	1.61	.71	2.51

T-Test

One-Sample Statistics

	N	Mean	Std. Deviation	Std. Error Mean
Study Hours	668	16.61	11.849	.458

One-Sample Test

	Test Value = 16					
					95% Confidence Interval of the Difference	
	t	df	Sig. (2-tailed)	Mean Difference	Lower	Upper
Study Hours	1.321	667	.187	.61	-.29	1.51

T-Test

One-Sample Statistics

	N	Mean	Std. Deviation	Std. Error Mean
Study Hours	668	16.61	11.849	.458

One-Sample Test

	Test Value = 17					
					95% Confidence Interval of the Difference	
	t	df	Sig. (2-tailed)	Mean Difference	Lower	Upper
Study Hours	-.860	667	.390	-.39	-1.29	.51

T-Test

One-Sample Statistics

	N	Mean	Std. Deviation	Std. Error Mean
Study Hours	668	16.61	11.849	.458

One-Sample Test

	Test Value = 18					
					95% Confidence Interval of the Difference	
	t	df	Sig. (2-tailed)	Mean Difference	Lower	Upper
Study Hours	-3.042	667	.002	-1.39	-2.29	-.49

The sample mean and sample size are the same for each case, that is, the sample mean hours spent studying is 16.61 and the sample size is n = 668. If the 0.05 level of significance is used, the critical Z for i. and iv. Is 1.645. For 0.05 significance in ii. and iii., the critical Z is plus or minus 1.96.

From the four tests, the results are as follows

- **i.** **$Z = 3.502 > +1.645$ Reject H0 and accept H1. ie. $\mu > 15$**
- **ii.** **$Z = 1.321$ and between $\pm$ 1.96. Do not reject H0, ie. $\mu = 16$**
- **iii.** **$Z = -0.860$ and between $\pm$ 1.96. Do not reject H0, ie. $\mu = 17$**
- **iv.** **$Z = -3.042 < -1.645$ Reject H0 and accept H1, ie. $\mu < 18$**

From these tests, what can be concluded is that study hours for all undergraduates are greater and 15 hours and less than 18 hours. That is, for i. and iv, the null hypothesis can be rejected. However, there is not evidence that mean study hours differ from either 16 or 17 hours. That is, neither of these two claims can be rejected, although both could not be

true. This latter conclusion demonstrates Type II error. That is, for these two hypotheses, both of which cannot be true, neither can be rejected.

In conclusion it appears that mean weekly study hours are between 15 and 18 hours for all undergraduates. The sample mean is 16.61, but in order to pin down a more exact value for the mean, a larger sample size would be necessary.