Social Studies 201
Winter 2001
Answers to Problem Set 5

1. (a) **Diversity Fundamental**. Let μ be the true mean response to the question concerning whether diversity is fundamental. A sample size of $n = 18$ cases is small, so the sample mean $\bar{X}$ has a sampling distribution which is a t-distribution with mean μ, standard deviation $s/\sqrt{n}$, and $d = n-1 = 17$ degrees of freedom. Note that this assumes that the distribution of responses for all undergraduates is normal, an assumption that is unlikely to hold. However, it is preferable to use the t-distribution to using the normal distribution for the sampling distribution of $\bar{X}$, so that the interval width is not underestimated. The 90% interval estimate is

$$\begin{aligned}\bar{X} \pm Z\frac{\sigma}{\sqrt{n}} &= 4.17 \pm 1.740 \; \frac{1.25}{\sqrt{18}} \\ &= 4.17 \pm 1.740 \; \frac{1.25}{4.243} \\ &= 4.17 \pm 1.740 \times 0.295 \\ &= 4.17 \pm 0.51\end{aligned}$$

or (3.66 , 4.68).
The 98% interval estimate is

$$\begin{aligned}\bar{X} \pm Z\frac{\sigma}{\sqrt{n}} &= 4.17 \pm 2.567 \; \frac{1.25}{\sqrt{18}} \\ &= 4.17 \pm 2.567 \; \frac{1.25}{4.243} \\ &= 4.17 \pm 2.567 \times 0.295 \\ &= 4.17 \pm 0.76\end{aligned}$$

or (3.41 , 4.93).
Fund Festivals. In this case, μ is the true mean response to the fund festivals question for all undergraduates. The method is the same, with the same t-values and n, but $\bar{X} = 3.22$ and $s = 1.40$. The 90% and 98% interval estimates are (2.65 , 3.79) and (2.37 , 4.07).

(b) For sample size,

$$n = \left(\frac{Z\sigma}{E}\right)^2.$$

Since the sample size is usually large, the sampling distribution of $\bar{X}$ has a normal distribution and Z=1.645 for 90% confidence. $E = 0.1$ and s can be used as an estimate for σ. Since the two questions have different standard deviations, in order to ensure that a large enough sample size is obtained, use the larger of the two standard deviations. Using these values,

$$n = \left(\frac{1.645 \times 1.40}{0.1}\right)^2 = 23.03^2 = 530.4.$$

A random sample of size $n = 531$ would provide the required precision (assuming that the standard deviation for the whole population is not any larger than 1.40).

(c) Let μ be the true mean response to the funding festivals question for undergraduates. Since the question asks whether the average response differs from 3, and no direction is noted, this is a two tailed test and the hypotheses are:

$$H_0 \ : \ \mu = 3$$

$$H_1 \ : \ \mu \neq 3$$

The test statistic is $\bar{X}$ and since $n = 18$, $\bar{X}$ has a t-distribution with mean μ, standard deviation $s/\sqrt{n}$, and $d = n-1 = 17$ degrees of freedom. At $\alpha = 0.05$ significance, the critical $t = 2.110$ and H_0 is rejected for all $t < -2.110$ or $t > +2.110$. If the sample mean yields a t-value between -2.110 and +2.110, the null hypothesis is not rejected.

$$\begin{aligned} t &= \frac{\bar{X} - \mu}{s/\sqrt{n}} \\ &= \frac{3.22 - 3}{1.40/\sqrt{18}} \\ &= \frac{0.22}{0.330} \\ &= 0.667 \end{aligned}$$

This value lies between the critical values of t and the null hypothesis cannot be rejected at the 0.05 level of significance. At the 0.05 level of significance the data do not provide sufficient evidence to conclude that the mean response of undergraduates to the funding festivals question is different from 3 or neutral.

For the diversity fundamental question, the method is the same but since the question asks whether the mean response exceeds 3, this is a one tailed test in the positive direction and the hypotheses are:

$$\begin{aligned} H_0 &: \mu = 3 \\ H_1 &: \mu > 3 \end{aligned}$$

The test statistic is $\bar{X}$ and since $n = 18$, $\bar{X}$ has a t-distribution with mean μ, standard deviation $s/\sqrt{n}$, and $d = n-1 = 17$ degrees of freedom. At $\alpha = 0.01$ significance, the critical $t = 2.567$ and H_0 is rejected for all $t > +2.567$.

$$\begin{aligned} t &= \frac{\bar{X} - \mu}{s/\sqrt{n}} \\ &= \frac{4.17 - 3}{1.25/\sqrt{18}} \\ &= \frac{1.17}{0.295} \\ &= 3.966 > 2.567 \end{aligned}$$

The t-value exceeds the critical value of $t = 2.567$ so the null hypothesis is rejected at the 0.01 level of significance. At the 0.01 level of significance the data provide evidence that the mean response of undergraduates to the diversity fundamental question is greater than 3 or neutral. In this case, since the sample mean is associated with a very large t value, there is strong evidence from the data that undergraduates support (since greater than 3 means agree) the view that diversity is fundamental in multiculturalism.

2. (a) Let μ be the true value of the increase in the mean university student debt load over a year, for those with an increase in debt load. Since the contention is that the mean increased by over \$5,000 in the year, this is a one tailed test with hypotheses:

$$H_0 : \mu = 5,000$$

$$H_1 : \mu > 5,000$$

The sample size for those with an increase in debt is $38 + 39 + 68 + 22 = 167$. The test statistic is $\bar{X}$ and since $n = 167$ is large, $\bar{X}$ has a normal distribution with mean μ and standard deviation $\sigma/\sqrt{n}$. At $\alpha = 0.10$ significance, the critical $Z = 1.28$ and H_0 is rejected for all $Z > +1.28$.

$$\begin{aligned} Z &= \frac{\bar{X} - \mu}{s/\sqrt{n}} \\ &= \frac{5,275 - 5,000}{3,689/\sqrt{167}} \\ &= \frac{275}{285.463} \\ &= 0.963 < 1.28 \end{aligned}$$

and the null hypothesis is not rejected at the 0.10 level of significance. While the sample shows that the mean increase in student debt load, for those having increased debt, has increased more than \$5,000, the increase is not enough more than \$5,000 to conclude that the contention is true.

(b) Let p be the true proportion of students that had no increase in debt load. Since $n = 574$ is large, the sample proportion $\hat{p}$ is normally distributed with mean p and standard deviation $\sqrt{pq/n}$, by the normal approximation to the binomial. Since the claim is that over two-thirds of students have no increase, this is a one-tailed test in the positive direction, with hypotheses

$$H_0 : p = 0.667$$

$$H_1 : p > 0.667$$

At $\alpha = 0.05$ significance, the critical $Z = 1.645$ and H_0 is rejected for all $Z > +1.645$. From Table 2, the sample proportion of students who reported no increase in debt load was $\hat{p} = (397 + 10)/574 = 407/574 = 0.709$.

$$
\begin{aligned}
Z &= \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} \\
&= \frac{0.709 - 0.667}{\sqrt{\frac{0.667 \times 0.333}{574}}} \\
&= \frac{0.709 - 0.667}{\sqrt{0.0003869}} \\
&= \frac{0.042}{0.01967} \\
&= 2.135 > 1.645
\end{aligned}
$$

and the data from the sample provides sufficient evidence to reject the null hypothesis at the 0.05 level of significance. The data from the sample shows that more than two-thirds of students had no increase in debt load and the claim by the politician from the North West Party appears to be correct – based on these data and at the 0.05 level of significance.

(c) The two claims appear to be contrary to each other, but the claims focus on somewhat different issues. According to these data, the politician is correct, that over two-thirds of students did not experience an increase in their debt load. However, for those who did have an increase in debt load, the increase was considerable – around and average of \$5,000 or more. The first test shows that the hypothesis of an increase in mean debt load of above \$5,000 cannot be supported; the sample does show an mean increase of more than \$5,000 and there is no evidence that the mean increase in debt load was less than \$5,000. So the result is that a large proportion of students did not have an increase in their debt load, but for those whose debt load increased, the mean increase appears to have been around \$5,000 over one year.

3. (a) Let p be the true proportion who express support for any particular opinion issue asked in the survey. The sample sizes of 200 to 2000 here are all reasonably large, so the sample proportions are normally distributed with mean p and standard deviation $\sqrt{pq/n}$, by the normal approximation to the binomial. Decima Express does not provide the confidence level, but the most common level used in opinion polls of this sort is 19 in 20 times or the 95% interval estimate. For this confidence level, $Z = 1.96$. By using $p = q = 0.5$, the widest possible interval for any of the proportions is obtained. Using these values, the interval estimates of the proportion of residents of each area who express support for any particular view in Manitoba/Saskatchewan (each with a sample size of $n = 200$) are:

$$\begin{aligned}\hat{p} \pm Z\sqrt{\frac{pq}{n}} &= \hat{p} \pm 1.96\sqrt{\frac{0.5 \times 0.5}{200}} \\ &= \hat{p} \pm 1.96\sqrt{0.00125} \\ &= \hat{p} \pm (1.96 \times 0.0354) \\ &= \hat{p} \pm 0.0693\end{aligned}$$

and the interval estimate is $\hat{p} \pm 0.069$ or plus or minus 6.9%, as stated in the Decima Express Methodology.

For Canada as a whole, the sample size of $n = 2000$ is considerably larger and the intervals are:

$$\begin{aligned}\hat{p} \pm Z\sqrt{\frac{pq}{n}} &= \hat{p} \pm 1.96\sqrt{\frac{0.5 \times 0.5}{2000}} \\ &= \hat{p} \pm 1.96\sqrt{0.000125} \\ &= \hat{p} \pm (1.96 \times 0.011803) \\ &= \hat{p} \pm 0.0219\end{aligned}$$

and the interval estimate is $\hat{p} \pm 0.022$ or plus or minus 2.2%, as stated in the Decima Express Methodology.

(b) For estimation of a proportion,

$$n = \left(\frac{Z}{E}\right)^2 pq.$$

For the 90% confidence, $Z = 1.645$. From the question, the accuracy required is an interval of 2 percentage points wide. This is equivalent to plus or minus 1 percentage point, or $E = 0.01$, and the largest possible sample size for this Z and E are given by $p = q = 0.5$. The required sample size is

$$n = \left(\frac{1.645}{0.01}\right)^2 \times 0.5 \times 0.5 = 164.5^2 \times 0.25 = 6,765.1$$

The required sample size is $n = 6,766$.

What is missing from the Decima Express Methodology is a statement of the confidence level used. As noted in class, it is meaningless to report sampling error, or what Decima Express calls an error interval, without reporting what confidence level is associated with the interval. Each different confidence level yields a different interval, so to be accurate, a researcher must report the confidence level used to obtain that particular interval. In this case, it appears that Decima Express has used the 95% level of confidence, but this is not stated in the methodology table.

(c) Let $\hat{p}$ be the true proportion of residents of Manitoba/Saskatchewan who support reducing the government debt. The sample proportion is $\hat{p} = 38/138 = 0.275$. The interval estimate is

$$\begin{aligned} \hat{p} \pm Z\sqrt{\frac{pq}{n}} &= \hat{p} \pm 2.575\sqrt{\frac{0.275 \times 0.725}{138}} \\ &= \hat{p} \pm 2.575\sqrt{0.000144} \\ &= \hat{p} \pm (2.575 \times 0.0380) \\ &= \hat{p} \pm 0.0979 \end{aligned}$$

Since $\hat{p} = 0.275$, the confidence interval is 0.275 ± 0.098 or $(0.177\ ,\ 0.373)$. This interval is wider than the ± 0.069 reported by Decima for two reasons. First, the confidence level is 99% rather than the 95% that Decima apparently used. In order to obtain an interval associated with the larger confidence level, more of the sampling distribution of $\hat{p}$ must be included, and this means a large Z value. Second, the sample size reported for Manitoba/Saskatchewan in Table 3 is only 138, rather than 200. Apparently Decima did not find 200 respondents in these two provinces or some respondents did not answer this particular question. As a result of these two considerations, the interval estimate here is wider than that reported by the Decima Express Methodology. Note that this is the case even though $\hat{p} = 0.275$ and $\hat{q} = 0.725$ were used in $\sqrt{\frac{pq}{n}}$ rather than $p = q = 0.5$, thus producing a smaller product for p times q. But this smaller value is dominated by the larger t asnd smaller n in the formula.