Social Studies 201
Fall 2004

Answers to Problem Set 5

1. **Hours worked for Saskatchewan and Canadian adults**

   (a) i. **Males**. Let μ be the true mean annual hours worked at jobs for all Saskatchewan male workers. From Table 1, for males the sample size is $n = 1,002$ and the test statistic is $\bar{X}$. Given this large sample size, $\bar{X}$ has a normal distribution with mean μ and standard deviation $\sigma/\sqrt{n}$, where the sample standard deviation s is used as an estimate of σ. For 92% confidence level, the appropriate Z value is 1.75 (46% or 0.4600 in A area is associated with $Z = 1.75$). The 92% interval estimate is

$$\begin{aligned}\bar{X} \pm Z\frac{\sigma}{\sqrt{n}} &= 2,334 \pm 1.75\ \frac{683}{\sqrt{1,002}} \\ &= 2,334 \pm 1.75\ \frac{683}{31.654} \\ &= 2,334 \pm 1.75 \times 21.577 \\ &= 2,334 \pm 37.759\end{aligned}$$

   Rounded to the neares hour, is is from 2,296 to 2,372 hours, or $(2,296\ ,\ 2,372)$ hours.

   ii. **Females**. For this part, μ is the true mean annual hours worked at jobs for all Saskatchewan male workers. From Table 1, the sample size for females is $n = 738$ and this is a large sample size, so the procdure is the same as in part i. The 92% interval estimate is

$$\begin{aligned}\bar{X} \pm Z\frac{\sigma}{\sqrt{n}} &= 1,975 \pm 1.75\ \frac{458}{\sqrt{738}} \\ &= 1,975 \pm 1.75\ \frac{458}{27.166} \\ &= 1,975 \pm 1.75 \times 16.589 \\ &= 1,975 \pm 29.504\end{aligned}$$

The interval is from 1,945 hours to 2,005 hours, rounded to the nearest hour, or $(2,296\ ,\ 2,372)$ hours.

(b) For estimation of a mean, the formula is

$$n = \left(\frac{Z\sigma}{E}\right)^2$$

where E is the accuracy required. For 95% confidence, $Z = 1.96$. Since the population standard deviation σ is unknown, the sample standard deviation is used to provide an estimate of this. Use the largest of the three standard deviations in Table 1 in order to obtain a large enough sample size. For $E = \pm 50$, the required sample size is

$$\begin{aligned} n &= \left(\frac{Z\sigma}{E}\right)^2 \\ &= \left(\frac{1.96 \times 683}{50}\right)^2 \\ &= (26.774)^2 \\ &= 716.83 \end{aligned}$$

The required sample size to achieve an accuracy of plus or minus 50 hours, with 95% confidence, is $n = 717$.

For $E = \pm 25$ hours, the method is identical and the sample size required is

$$\begin{aligned} n &= \left(\frac{Z\sigma}{E}\right)^2 \\ &= \left(\frac{1.96 \times 683}{25}\right)^2 \\ &= (53.547)^2 \\ &= 2,867.30 \end{aligned}$$

The required sample size to achieve an accuracy of plus or minus 25 hours, with 95% confidence, is $n = 2,868$.

(c) i. **Males**. Let μ be the true mean annual hours worked at jobs for all Saskatchewan male workers. From Table 1, for males the sample size is $n = 1,002$ and the test statistic is $\bar{X}$. Given this large sample size, $\bar{X}$ has a normal distribution with mean μ and standard deviation $\sigma/\sqrt{n}$, where the sample standard deviation s is used as an estimate of σ. The test is a one-tailed test since the question is whether the Saskatchewan mean **exceeds** the Canadian mean of 2,169 hours. The hypotheses are:

$$H_0 : \mu = 2,169$$

$$H_1 : \mu > 2,169$$

For a normal distribution, a one-tailed test at the $\alpha = 0.01$ significance level has a critical value of $Z = 2.33$. H_0 is rejected for all greater than $Z > +2.33$. The Z-value for the test is

$$\begin{aligned} Z &= \frac{\bar{X} - \mu}{s/\sqrt{n}} \\ &= \frac{2,234 - 2,169}{683/\sqrt{1,002}} \\ &= \frac{65}{683/31.654} \\ &= \frac{65}{21.577} \\ &= 3.012 \end{aligned}$$

Since this Z-value is greater than +2.33, the the null hypothesis is rejected at the 0.01 level of significance. At the 0.01 level of significance this sample provides evidence that the mean annual hours that Saskatchewan male workers work at jobs exceeds the mean for all Canadian male workers.

ii. **Females**. For this part, let μ be the true mean annual hours that Saskatchewan females work at jobs. The method is the same as in part i., and again the test is a one-tailed test, so the hypotheses are

$$H_0 : \mu = 1,917$$

$$H_1 : \mu > 1,917$$

The Z value is

$$\begin{aligned} Z &= \frac{\bar{X} - \mu}{s/\sqrt{n}} \\ &= \frac{1,975 - 1,917}{458/\sqrt{738}} \\ &= \frac{58}{458/27.166} \\ &= \frac{58}{16.859} \\ &= 3.440 \end{aligned}$$

Since $Z = 3.44$ is greater than the critical value of $Z = +2.33$ for a one-tailed test at the 0.01 level of significance, the null hypothesis is rejected and the alternative hypothesis accepted. At the 0.01 level of significance, the data from Table 1 provides strong evidence that the mean hours worked for all Saskatchewan females exceeds the mean for their counterparts across Canada.

2. **Annual wages and salaries**.

For this question, the wages and salaries of eight workers are given. In order to obtain interval estimates and test the hypothesis, it is necessary to compute mean and standard deviation of wages and salaries for these eight workers. Let X be the wages and salaries for these workers. The values of X are 55, 39, 5, 16, 12, 22, 58, and 49 thousand dollars. The mean, $\bar{X} = 32.0$ and the standard deviation is $s = 20.798$, each in thousands of dollars.

(a) For these interval estimates, the population value to be estimated is μ, the true mean annual wages and salaries for all Saskatchewan workers. The point estimate is $\bar{X} = 32$ thousand dollars and, since $n = 8$ is very small, the sample mean has a t-distribution with mean μ, standard deviation $s/\sqrt{n}$, and $n - 1 = 8 - 1 = 7$ degrees of freedom. For a 90% interval, the t-value for 7 degrees of freedom is 1.895; for 99% confidence and 7 degrees of freedom, $t = 3.500$.

The 90% interval estimate is

$$\begin{aligned}
\bar{X} \pm t\frac{\sigma}{\sqrt{n}} &= 32 \pm 1.895 \ \frac{20.798}{\sqrt{8}} \\
&= 32 \pm 1.895 \ \frac{20.798}{2.828} \\
&= 32 \pm 1.895 \times 7.353 \\
&= 32 \pm 13.934
\end{aligned}$$

Rounded to the nearest thousand dollars, the 90% interval estimate for the wages and salaries of Saskatchewan workers is from 18.1 to 45/9 thousand dollars, or (18.1 , 45.9) thousand dollars.
The 99% interval estimate is

$$\begin{aligned}
\bar{X} \pm t\frac{\sigma}{\sqrt{n}} &= 32 \pm 3.500 \ \frac{20.798}{\sqrt{8}} \\
&= 32 \pm 3.500 \ \frac{20.798}{2.828} \\
&= 32 \pm 3.500 \times 7.353 \\
&= 32 \pm 25.736
\end{aligned}$$

The corresponding 99% interval is or (6.3 , 57.7) thousand dollars.

(b) Again, the parameter to be considered is μ, the true mean wages and salaries for all Saskatchewan workers. The question is whether this is **lower** than their Canadian counterparts. As a result, the test is a one-tailed test of whether μ is less than the Canadian mean or not. The hypotheses are:

$$H_0 \ : \ \mu = 35.0$$

$$H_1 \ : \ \mu < 35.0$$

The statistic used to test the hypotheses is $\bar{X} = 32.0$. Since the sample size is only $n = 8$ for this sample, the t-distribution is used. In order to use the t-distribution, the distribution of the population should be close to normally distributed – while there is no assurance that wages of Saskatchewan workers are normally distributed, it has to be assumed that they are. The sample mean

$\bar{X}$ has a t-distribution with mean μ, standard deviation $s/\sqrt{n}$, and $n-1 = 8-1 = 7$ degrees of freedom. For 7 d.f., $\alpha = 0.05$ significance, and a one-tailed t test, the critical $t = -1.895$ and H_0 is rejected for all $t < -1.895$.

$$\begin{aligned} t &= \frac{\bar{X}-\mu}{s/\sqrt{n}} \\ &= \frac{32.0-35.0}{20.798/\sqrt{8}} \\ &= \frac{-3.0}{7.353} \\ &= -0.408 \end{aligned}$$

This t-value is to the right of -1.895, and this t-value is not in the region for rejection of the null hypothesis. As a result, the null hypothesis that the mean wages and salaries of Saskatchewan workers are less than those for all Canadian workers cannot be rejected at the 0.05 level of significance. At 0.05 significance, there is no conclusive evidence that Saskatchewan workers are paid less than their counterparts across Canada.

(c) The interval estimates of part a. are very wide, from 18.1 to 45.9 thousand dollars and 6.3 to 57.7 thousand dollars. Each of these intervals is wide enough to contain the Canadian mean of 35.0 thousand dollars. Since the true Saskatchewan mean could be anywhere within the interval, and the interval contains the Canadian mean, the sample does not provide evidence that the Saskatchewan and Canadian means differ.

The hypothesis test confirms this. In part b., there is a direct test of whether the Saskatchewan and Canadian mean wages and salaries are the same or whether the Saskatchewan mean is less than the Canadian mean. The null hypothesis of no difference in means cannot be rejected. Again, the sample does not provide sufficient evidence to conclude that Saskatchewan workers are paid less than their Canadian counterparts.

The problem here is that the sample size of $n = 8$ is so small that the data from the sample cannot be used to distinguish pay levels of Saskatchewan workers from the Canadian average. The sample mean for Saskatchewan is 32 thousand dollars, less than the Canadian average of 35 thousand dollars. But with a standard deviation of close to 21 thousand dollars and a sample size of only 8, the sample does not really show that the Saskatchewan average is less than the Canadian average. Thus the results from a. and b. are consistent.

In terms of errors, there is a possibility of a Type II error in part b. Type II error is the error of failing to reject a false hypothesis. The null hypothesis that the Saskatchewan mean pay equals that for Canada as a whole is likely incorrect – it is very unlikely that the mean for Saskatchewan would be exactly equal to the mean for all Canada. But the null hypothesis was not rejected in part b., so there was undoubtedly Type II error. Again, the problem is the small sample size and large standard deviation for wages and salaries – this sample does not provide strong enough results to distinguish Saskatchewan pay levels from those of Canada as a whole.

Other errors could be associated with whether or not the sample is random and whether the sample standard deviation of $s = 20.798$ thousand dollars is a good estimate of the true standard deviation of wages and salaries for all Saskatchewan workers. But these are problems for all samples. The main problem here is the small sample size, very likely leading to Type II error.

3. **Opinions about selling Petro-Canada**

(a) i. **Canadian adults**. Let p be the true proportion of all Canadian adults who agree (strongly or somewhat) that Petro Canada should not be sold. The sample in Table 2 indicates that $48 + 27 = 75\%$ or $\hat{p} = 0.75$ of Canadian adults in the sample agreed that it should not be sold. From this, the proportion who did not agree is $\hat{q} = 1 - 0.75 = 0.25$. Since $n = 1,057$ is large ($n = 1,057 > 5/0.25 = 20$), the sample proportion $\hat{p}$ is normally distributed with mean p and standard deviation $\sqrt{pq/n}$. For the 97% interval estimate,

$Z = \pm 2.17$. By using $p = q = 0.5$, the widest possible interval for any of the proportions is obtained. Alternatively, $\hat{p} = 0.25$ and $\hat{q} = 1 - \hat{p} = 1 - 0.25 = 0.75$ could be used for p and q in the estimate of the standard deviation of $\hat{p}$. The interval estimates are:

$$\begin{aligned} \hat{p} \pm Z\sqrt{\frac{pq}{n}} &= \hat{p} \pm 2.17\sqrt{\frac{0.25 \times 0.75}{1,067}} \\ &= \hat{p} \pm 2.17\sqrt{0.000176} \\ &= \hat{p} \pm (2.17 \times 0.0133) \\ &= \hat{p} \pm 0.0288 \\ &= 0.75 \pm 0.03 \end{aligned}$$

To the nearest percentage, the interval is 75% plus or minus 3 percentage points, or from 0.72 to 0.78. The 97% interval estimate for married men is from 72% to 78%.

ii. **Saskatchewan/Manitoba adults**. For adults in Saskatchewan and Manitoba, the method is the same and with a sample size of $n = 100$, again $\hat{p}$ is normally distributed and $Z = \pm 2.17$ for 97% confidence. $\hat{p} = 0.72$ and $\hat{q} = 1 - \hat{p} = 1 - 0.72 = 0.28$, and these are used to construct the interval. The interval is

$$\begin{aligned} \hat{p} \pm Z\sqrt{\frac{pq}{n}} &= \hat{p} \pm 2.17\sqrt{\frac{0.72 \times 0.28}{100}} \\ &= \hat{p} \pm 2.17\sqrt{0.002016} \\ &= \hat{p} \pm (2.17 \times 0.045) \\ &= \hat{p} \pm 0.097 \\ &= 0.72 \pm 0.10 \end{aligned}$$

or from 0.62 to 0.82. The 97% interval estimate for the proportion of adults in Saskatchewan and Manitoba who agree that Petro Canada should not be sold is from 62% to 82%.

(b) Let p be the true proportion of all Saskatchewan and Manitoba adults who agree that Petro Canada should not be sold. The question is to test whether this proportion is less than three-quarters, that is 0.75, or 75%. Since the question asks whether Saskatchewan and Manitoba are "less in agreement," this is a one-tailed test in the negative direction. The null hypothesis is an equality so the hypotheses are:

$$H_0 \;:\; p = 0.75$$

$$H_1 \;:\; p < 0.75$$

The sample size for the Saskatchewan and Manitoba sample is $n = 100$ and the test statistic is $\hat{p}$, where $\hat{p} = 0.72$, $\hat{q} = 1 - \hat{p} = 1 - 0.72 = 0.28$. To ensure this is a large sample size, n divided by the smaller of p or q gives $5/0.28 = 17.9$ and $n = 100 > 17.9$ so this is a large sample size and $\hat{p}$ has a normal distribution with mean p and standard deviation $\sqrt{pq/n}$. At $\alpha = 0.10$ significance, with a one-tailed test, the critical $Z = 1.28$. When $Z < -1.28$, the null hypothesis can be rejected but when $Z > -1.28$, the null hypothesis is not rejected.

$$\begin{aligned} Z &= \frac{\hat{p} - p}{\sqrt{pq/n}} \\ &= \frac{0.72 - 0.75}{\sqrt{(0.75 \times 0.25)/100}} \\ &= \frac{-0.03}{0.043} \\ &= -0.693 \end{aligned}$$

Since this Z-value is to the right of -1.28, the the null hypothesis is not rejected at the 0.10 level of significance. At the 0.10 level of significance the sample results do not provide sufficient evidence to conclude that Saskatchewan and Manitoba residents are less in agreement with not selling Petro Canada.

(c) From the interval estimates and the hypothesis test, there is not sufficient evidence to conclude that adults in Saskatchewan and Manitoba differ from other Canadians on this issue. While only $36 + 36 = 72\%$ of the 100 adults sampled in Saskatchewan and

Manitoba agree with not selling, and this is less than the $48+27 = 75\%$ with the same view across Canada, the results do not differ sufficiently to conclude the two groups are different. The interval estimates of part (a) show that the intervals for Saskatchewan and Manitoba is from 62% to 82% in agreement; for the Canadian sample the interval is from 72% to 78%. The interval for Saskatchewan actually overlaps the whole interval for Canada, so it cannot be concluded that there is any difference between the two groups in the the true proportions in agreement. The hypothesis test of part (b) tests this directly and the conclusion is that the two groups do not differ. While there is a possibility of Type II error, and the two groups differ, the sample proportions do not differ sufficiently to make this conclusion.

(d) For estimation of any proportion, the sample size required to achieve the specified accuracy E is

$$n = \left(\frac{Z}{E}\right)^2 pq.$$

19 times out of twenty is equivalent to 95% confidence, since $19/20 = 0.95$. For 95% confidence, $Z = \pm 1.96$. The precision stated is ± 3.1 percentage points, so $E = 0.031$ as a proportion. One way to verify the statement is to use this Z and E, along with $p = q = 0.5$ to see how close this comes to the stated $n = 1.057$. For these values,

$$n = \left(\frac{Z}{E}\right)^2 pq = \left(\frac{1.96}{0.031}\right)^2 0.5 \times 0.5 = 999.4$$

While this a bit less than the sample size of 1,057 stated in the question, it is close. A sample of approximately 1,000 respondents is required to achieve an accuracy of plus or minus 3.1 percentage points, 19 times out of twenty.

For Saskatchewan and Manitoba, the easiest way to obtain the margin is to use the formula for interval estimates. For 95% confidence, $Z = \pm 1.96$, and using $p = q = 0.5$ with a sample size of $n = 100$, gives intervals such as

$$\hat{p} \pm Z\sqrt{\frac{pq}{n}} \quad = \quad \hat{p} \pm 1.96\sqrt{\frac{0.5 \times 0.5}{100}}$$

$$\begin{aligned} &= \hat{p} \pm 1.96\sqrt{0.0025} \\ &= \hat{p} \pm (1.96 \times 0.05) \\ &= \hat{p} \pm 0.098 \end{aligned}$$

Rounding this off, the comparable interval margin of error for a sample size of 100, 19 times out of 20, is plus or minus 10 percentage points, that is ± 0.10.

Computer problem

1. Views about gay and lesbian couples – statement V4.

a. i. 95% interval estimates for V4.

Case Processing Summary

	Cases					
	Valid		Missing		Total	
	N	Percent	N	Percent	N	Percent
V4 Gay and Lesbians Married	695	98.3%	12	1.7%	707	100.0%

Descriptives

			Statistic	Std. Error
V4 Gay and Lesbians Married	Mean		3.06	.052
	95% Confidence Interval for Mean	Lower Bound	2.96	
		Upper Bound	3.16	
	5% Trimmed Mean		3.07	
	Median		3.00	
	Variance		1.880	
	Std. Deviation		1.371	
	Minimum		1	
	Maximum		5	
	Range		4	
	Interquartile Range		2.00	
	Skewness		-.164	.093
	Kurtosis		-1.150	.185

The mean of V4 is 3.06, almost exactly the middle response (neutral) between the extremes of 1, strongly disagree, and 5, strongly agree. The 95% confidence interval is from 2.96 to 3.16, from just under the neutral response of 3 to a little above the neutral response. If these represent a more or less random sample of undergraduates, it appears that the mean response for undergraduates is neutral or 3, or very close to this.

a. ii. 98% interval estimates for V4, classified by sex of respondent

Case Processing Summary

	SEX SEX OF RESPONDENT	Cases					
		Valid		Missing		Total	
		N	Percent	N	Percent	N	Percent
V4 Gay and Lesbians Married	1 MALE	260	99.2%	2	.8%	262	100.0%
	2 FEMALE	435	97.8%	10	2.2%	445	100.0%

Descriptives

	SEX SEX OF			Statistic	Std. Error
V4 Gay and Lesbians Married	1 MALE	Mean		2.84	.087
		98% Confidence Interval for Mean	Lower Bound	2.64	
			Upper Bound	3.05	
		5% Trimmed Mean		2.82	
		Median		3.00	
		Variance		1.971	
		Std. Deviation		1.404	
		Minimum		1	
		Maximum		5	
		Range		4	
		Interquartile Range		3.00	
		Skewness		.005	.151
		Kurtosis		-1.254	.301
	2 FEMALE	Mean		3.19	.064
		98% Confidence Interval for Mean	Lower Bound	3.04	
			Upper Bound	3.34	
		5% Trimmed Mean		3.21	
		Median		3.00	
		Variance		1.785	
		Std. Deviation		1.336	
		Minimum		1	
		Maximum		5	
		Range		4	
		Interquartile Range		2.00	
		Skewness		-.255	.117
		Kurtosis		-1.044	.234

b. ii. Males and females appear to have somewhat different responses to the statement about gay and lesbian couples. The mean response to this statement for males is 2.84, with a 98% confidence interval from 2.64 to 3.05. This implies that male responses are slightly below the neutral response of 3, and appear to be on the disagree side of the statement. For females, the mean is 3.19, and the 98% interval estimate is from 3.04 to 3.34. The mean and the associated interval estimate imply that the mean female response is on the agree side. That is, given a high confidence level of 98%, the interval estimate for female responses is entirely on the agree side (above 3).

a. iii. Interval estimates for V4, classified by provincial political preference, PV.

Case Processing Summary

		Cases					
	PV provincial political preference	Valid N	Valid Percent	Missing N	Missing Percent	Total N	Total Percent
V4 Gay and Lesbians Married	1 Liberal	97	100.0%	0	.0%	97	100.0%
	2 NDP	174	100.0%	0	.0%	174	100.0%
	3 Conservative	99	100.0%	0	.0%	99	100.0%
	4 None	165	98.8%	2	1.2%	167	100.0%

Descriptives

	PV provincial			Statistic	Std. Error
V4 Gay and Lesbians Married	1 Liberal	Mean		2.78	.145
		90% Confidence Interval for Mean	Lower Bound	2.54	
			Upper Bound	3.02	
		5% Trimmed Mean		2.76	
		Median		3.00	
		Variance		2.026	
		Std. Deviation		1.423	
		Minimum		1	
		Maximum		5	
		Range		4	
		Interquartile Range		3.00	
		Skewness		.104	.245
		Kurtosis		-1.181	.485
	2 NDP	Mean		3.35	.105
		90% Confidence Interval for Mean	Lower Bound	3.18	
			Upper Bound	3.52	
		5% Trimmed Mean		3.39	
		Median		4.00	
		Variance		1.905	
		Std. Deviation		1.380	
		Minimum		1	
		Maximum		5	
		Range		4	
		Interquartile Range		3.00	
		Skewness		-.425	.184
		Kurtosis		-.993	.366
	3 Conservative	Mean		2.61	.138
		90% Confidence Interval for Mean	Lower Bound	2.38	
			Upper Bound	2.84	
		5% Trimmed Mean		2.56	
		Median		3.00	
		Variance		1.894	
		Std. Deviation		1.376	
		Minimum		1	
		Maximum		5	
		Range		4	
		Interquartile Range		3.00	
		Skewness		.146	.243
		Kurtosis		-1.355	.481
	4 None	Mean		3.25	.103
		90% Confidence Interval for Mean	Lower Bound	3.08	
			Upper Bound	3.42	
		5% Trimmed Mean		3.28	
		Median		3.00	
		Variance		1.737	
		Std. Deviation		1.318	
		Minimum		1	
		Maximum		5	
		Range		4	
		Interquartile Range		2.00	
		Skewness		-.323	.189
		Kurtosis		-.975	.376

a. iii. The mean responses for those who favour each of the three political parties, or none, are as follows.

Party	Mean	90% interval
Liberal	2.78	(2.54, 3.02)
NDP	3.35	(3.18, 3.52)
Conservative	2.61	(2.38, 2.84)
None	3.25	(3.08, 3.42)

From the means and 90% interval estimates, those who support the NDP or no political party are much more strongly in favour of recognizing gay and lesbian couples as married. The means response for those who identify with each of these parties are well above the neutral response of three and the 90% interval estimates do not cross the neutral response of 3. Those who identify with a Conservative party express the greatest disagreement with recognition of gay and lesbian couples as married, with those supporting the Liberals close to the Conservative group. The interval estimate for the Conservative group does not cross 3, so it can be concluded (at 90% confidence) that Conservative supporters disagree with this. The Liberal supporters are a little closer to neutral, but the interval just crosses a neutral response of 3, so views of Liberal supporters tend to be on the disagree side about this statement.

2. One-Sample T tests.

The test is to determine whether the hours per week at the job for UR undegraduates (defined as μ) differ from those for undergraduates across Canada. Since there is no indication of a direction, the question is merely whether the means are the same or different, the alternative hypothesis is a two-directional test. The hypotheses are

Ho: $\mu = 17.2$

H1: $\mu \neq 17.2$

One-Sample Statistics

	N	Mean	Std. Deviation	Std. Error Mean
JOBHOURS HOURS PER WEEK AT JOB - F98	396	20.12	11.764	.591

One-Sample Test

	Test Value = 17.2					
					95% Confidence Interval of the Difference	
	t	df	Sig. (2-tailed)	Mean Difference	Lower	Upper
JOBHOURS HOURS PER WEEK AT JOB - F98	4.946	395	.000	2.92	1.76	4.09

The sample size is large, n=396, so the sample mean is normally distributed. At the 0.05 level of significance, the critical region is all Z < -1.96 or Z > +1.96. From the SPSS output,

the Z -value associated with the sample mean of 20.12 for UR undergraduates is 4.946. While listed as a t-value, since n>30, the sample is large so this t becomes a normal value or a Z-value. This Z exceeds +1.96, so the null hypothesis is rejected. The Z-value associated with the sample mean of 20.12 is well into the critical region, so this sample provides strong evidence that UR undergraduates differ from their Canadian counterparts.

ii. Let μ be the true mean income for the families/households of UR undergraduates. The question is whether this mean exceeds the mean income of households in each of the two provinces. In each case, the alternative hypothesis is thus a one-directional statement, that μ > provincial mean.

For Saskatchewan, the hypotheses are:

Ho: μ = 55.1 thousand dollars

H1: μ > 55.1 thousand dollars

One-Sample Statistics

	N	Mean	Std. Deviation	Std. Error Mean
INC income in thousands of dollars	617	65.46	43.888	1.767

The sample size is 617. This is very large, so once again the sample mean is normally distributed. If the 0.01 level of significance is adopted, the critical region is all Z > + 2.33.

One-Sample Test

	Test Value = 55.1					
					95% Confidence Interval of the Difference	
	t	df	Sig. (2-tailed)	Mean Difference	Lower	Upper
INC income in thousands of dollars	5.865	616	.000	10.36	6.89	13.83

The Z-value associated with the sample mean of 65.46 is Z = +5.865. This is greater than +2.33, so the null hypothesis is strongly rejected and the alternative hypothesis is accepted. There is strong evidence that the households of UR undergraduates have a mean income that exceeds the Saskatchewan mean (0.01 level of significance for this test).

For comparison with Ontario, the hypotheses are:

Ho: μ = 62.0 thousand dollars

H1: μ > 62.0 thousand dollars

One-Sample Statistics

	N	Mean	Std. Deviation	Std. Error Mean
INC income in thousands of dollars	617	65.46	43.888	1.767

One-Sample Test

	Test Value = 62					
					95% Confidence Interval of the Difference	
	t	df	Sig. (2-tailed)	Mean Difference	Lower	Upper
INC income in thousands of dollars	1.959	616	.051	3.46	-.01	6.93

The procedure for conducting the test is identical to that in part i.

In this case, Z=1.959, so the null hypothesis cannot be rejected at the 0.01 level of significance, as in i. However, if the 0.05 level of significance is used, the critical Z-value is +1.645. For this sample, the t-value is again a Z-value (since n>30) and Z=+1.959>+1.645. Thus the alternative hypothesis can be accepted at the 0.05 level of significance, but not at the 0.01 level of significance.

Comparing the results of i. and ii., the mean income for the sample is 65.46 thousand dollars, considerably greater than the mean for Saskatchewan (55.1 thousand dollars) but only a little greater than the mean for Ontario (62 thousand dollars). The sample size is the same (the 617 UR undergraduates sampled) and the standard deviation of the sample is the same, so it is no surprise that there is strong evidence for accepting the alternative hypothesis in I., but only weaker evidence in ii.

3. Four one-sample t tests. For each of the following tests, let μ be the true mean debt of UR undergraduates after the winter 1999 semester. The hypotheses are stated before each part. The sample data is the same for each test. A large sample size of n = 590 means that the sample mean is normally distributed and, once again, the t-values are really Z-values. Conclusions will be stated using the 0.05 significance level throughout. This means a critical region starting at -1.645 or +1.645 for the one-tailed test and at -1.96 and +1.96 for the two-tailed tests.

i. Hypotheses are

Ho: μ = 4 thousand dollars

H1: μ > 4 thousand dollars

One-Sample Statistics

	N	Mean	Std. Deviation	Std. Error Mean
DEBT2 Debt after Winter 1999	590	4612.88	8155.726	335.766

One-Sample Test

	Test Value = 4000					
					95% Confidence Interval of the Difference	
	t	df	Sig. (2-tailed)	Mean Difference	Lower	Upper
DEBT2 Debt after Winter 1999	1.825	589	.068	612.88	-46.56	1272.33

Z = +1.825 > +1.645, so reject Ho and accept H1. At 0.05 significance, conclude that debt exceed $4,000.

ii.

Ho: μ = 4,500

H1: $\mu \neq$ 4,500

One-Sample Statistics

	N	Mean	Std. Deviation	Std. Error Mean
DEBT2 Debt after Winter 1999	590	4612.88	8155.726	335.766

One-Sample Test

	Test Value = 4500					
					95% Confidence Interval of the Difference	
	t	df	Sig. (2-tailed)	Mean Difference	Lower	Upper
DEBT2 Debt after Winter 1999	.336	589	.737	112.88	-546.56	772.33

Z = 0.336 and this is between -1.96 and +1.96, so this sample mean is not in the critical region. Conclusion is to not reject the null hypothesis that the mean debt is $4,500. (0.05 significance level).

iii.

Ho: $\mu = 5{,}000$

H1: $\mu \neq 5{,}000$

One-Sample Statistics

	N	Mean	Std. Deviation	Std. Error Mean
DEBT2 Debt after Winter 1999	590	4612.88	8155.726	335.766

One-Sample Test

	Test Value = 5000					
					95% Confidence Interval of the Difference	
	t	df	Sig. (2-tailed)	Mean Difference	Lower	Upper
DEBT2 Debt after Winter 1999	-1.153	589	.249	-387.12	-1046.56	272.33

Z = -1.153 and again this is between the critical values of -1.96 and +1.96, so do not reject the null hypothesis that the mean debt if $5,000. (0.05 significance).

iv.

Ho: $\mu = 5{,}500$

H1: $\mu < 5{,}500$

One-Sample Statistics

	N	Mean	Std. Deviation	Std. Error Mean
DEBT2 Debt after Winter 1999	590	4612.88	8155.726	335.766

One-Sample Test

	Test Value = 5500					
					95% Confidence Interval of the Difference	
	t	df	Sig. (2-tailed)	Mean Difference	Lower	Upper
DEBT2 Debt after Winter 1999	-2.642	589	.008	-887.12	-1546.56	-227.67

Z = -2.642 < -1.96, so reject the null hypothesis. At the 0.05 level of significance, reject the hypothesis that the mean debt is $5,500 and conclude that it is less than $5,500.

Comparison:

From the above results, and using the 0.05 level of statistical significance, it can be concluded that the mean debt of UR undergraduates is greater than $4,000 and less than $5,500 (from parts I and iv). These conclusions are quite definite and strong, in that in i. and iv. the null hypotheses can each be rejected. While there is a possibility of Type I error, rejecting these hypotheses in error, the probability of this type of error is less than one chance in twenty (0.05).

But in tests ii. and iii., neither of the two hypotheses that the mean debt is $4,500 or $5,000 can be rejected. Here are two different null hypotheses, neither of which can be rejected. This illustrates two principles. First, it is not a good idea to accept the null hypothesis. As demonstrated here, there may be another null hypothesis that cannot be rejected and two different null hypotheses cannot each be accepted. So when not rejecting a null hypothesis, leave it at that; do not accept a null hypothesis, merely do not reject it if the Z or t is not in the critical region. This also illustrates Type II error, failing to reject a false hypothesis. Both the null hypotheses of ii and iii cannot be simultaneously true, but neither could be rejected. So there is Type II error in one or both of these conclusions.

In summary, it appears that debt is somewhere between $4,000 and $5,500, but exactly where is not clear. A larger sample would be required to attach a more specific value to the debt.