Sociology 405/805
Revised January 13, 2004

Measures of association in 2×2 tables using cross-products

For a table with two rows and two columns, there are three common measures of association using cross-products. These are the odds ratio α, Yule's Q, and the Pearson correlation coefficient, r. This handout introduces these three measures of association.

Using the notation of Table 1. In this table, n_{ij} represents the number of cases that are found in row i and column j. The marginal row totals are represented by n_{i+}, the marginal column totals by n_{+j}, and the grand total by n_{++} or simply by n. There are several measures of association which can be based on the **cross-products**, $n_{11}n_{22}$ and $n_{21}n_{12}$, and the relationship between these cross products. A good introduction and guide to these is contained in H. T. Reynolds, *Analysis of Nominal Data*, Sage Quantitative Applications in the Social Sciences Series, No. 7 (HA 33 R48, on reserve). Also see Albert M. Liebetrau, *Measures of Association*, Sage Quantitative Applications in the Social Sciences Series, No. 32 (HA 31.3 L53 1983).

	X		
Y	1	2	Totals
1	n_{11}	n_{12}	n_{1+}
2	n_{21}	n_{22}	n_{2+}
Totals	n_{+1}	n_{+2}	$n_{++} = n$

Table 1: General Format and Notation for a 2×2 Table

1. **The Odds Ratio** α. The odds or cross-product ratio can be written as

$$\hat{\alpha} = \frac{\frac{n_{11}}{n_{21}}}{\frac{n_{12}}{n_{22}}} = \frac{n_{11}n_{22}}{n_{21}n_{12}}.$$

For α, the minimum value is 0 when either or both n_{11} or n_{22} equals 0. The maximum value is $+\infty$ when either or both n_{21} or n_{12} equals 0.

One or other of these situations occurs when there is either perfect or complete association.

When there is no relationship between X and Y so that

$$\frac{n_{11}}{n_{21}} = \frac{n_{12}}{n_{22}}$$

then $\hat{\alpha} = 1$.

2. **Yule's Q**.

$$Q = \frac{n_{11}n_{22} - n_{12}n_{21}}{n_{11}n_{22} + n_{12}n_{21}} \tag{1}$$

In the case of no association or relationship, that is, $n_{11}/n_{21} = n_{12}/n_{22}$, then $n_{11}n_{22} = n_{12}n_{21}$ and $Q = 0$. $Q = 1$ if either n_{12} or n_{21} or both equal zero, that is, in the case of either complete or perfect association. Also

$$Q = \frac{\alpha - 1}{\alpha + 1}$$

and for a 2×2 table, $Q = \gamma$ (gamma).

3. **Pearson** r. For a 2×2 table, the Pearson coefficient of correlation, r, is given by the following:

$$r = \frac{n_{11}n_{22} - n_{12}n_{21}}{\sqrt{n_{1+}n_{2+}n_{+1}n_{+2}}}$$

For a 2×2 table, $r = V = \phi$ and $r^2 = \phi^2$.