Appendix J

The Chi Square Distribution

The $\chi^2$ distribution is an asymmetric distribution that has a minimum value of 0, but no maximum value. The curve reaches a peak to the right of 0, and then gradually declines in height, the larger the $\chi^2$ value is. The curve approaches, but never quite touches, the horizontal axis.

For each degree of freedom there is a different $\chi^2$ distribution. The mean of the chi square distribution is the degree of freedom and the standard deviation is twice the degrees of freedom. This implies that the $\chi^2$ distribution is more spread out, with a peak farther to the right, for larger than for smaller degrees of freedom. As a result, for any given level of significance, the critical region begins at a larger chi square value, the larger the degree of freedom.

Figure J.1 shows the shape of the distribution. The $\chi^2$ value is on the horizontal axis, with the probability for each $\chi^2$ value being represented by the vertical axis. The shaded area in the diagram represents the level of significance $\alpha$ shown in the table.

The $\chi^2$ table which follows gives $\chi^2$ values for selected levels of significance. All of the levels of significance shown represent areas in the right tail of the chi square distribution. The first page of the table shows $\chi^2$ values for the commonly used levels of significance. For example, if the $\alpha = 0.05$ level of significance is selected, and there are 7 degrees of freedom, the critical chi square value is 14.067. This means that for 7 degrees of freedom, there is exactly 0.05 of the area under the chi square distribution that lies to the right of $\chi^2 = 14.067$.

The second page of the table gives chi square values for the left end and the middle of the distribution. Again, the $\alpha$s across the top represent...
Figure J.1: The $\chi^2$ distribution
Figure J.1: The $\chi^2$ distribution

areas that lie to the right of the $\chi^2$ values shown in the table. For example, for 5 degrees of freedom, there is 0.95 of the area that lies to the right of $\chi^2 = 1.610$. The median $\chi^2$ value for 5 degrees of freedom is 4.352. That is, at $\alpha = 0.5$, and 5 degrees of freedom, 0.5 of the area under the curve lies to the right of $\chi^2 = 4.352$, with the other 0.5 to the left of this chi square value.

The chi square value on the second page of the table are not commonly used. However, they could be used when attempting to show how close a frequency distribution matches some hypothesized distribution. In this case, small values for the chi square statistic, or large significance levels, show a close match between the frequency distribution and the hypothesized values. Similarly, when examining cross classification of two variables, these smaller chi square values could be used to show that how close to exact independence the two variables are.

The chi square values in this table were generated by the SHAZAM program. The $\chi^2$ values are accurate to three decimal places.
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Table of the chi square distribution – Appendix J, p. 915