Math 9052B/4152B - Algebraic Topology
Winter 2015
Homework 4

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Due date: Wednesday March 11

Problem 1. (Variant of Hatcher #1.3.25) Let $X = \mathbb{R}^2 \setminus \{0\}$ and consider the action of the group $G = \mathbb{Z}$ on X generated by the homeomorphism $T \colon X \to X$ defined by $T(x, y) = (2x, \frac{y}{2})$.

(a) Show that this action is a "covering space action": each point in X has a neighborhood U such that the condition $U \cap gU \neq \emptyset$ implies $g = 0 \in \mathbb{Z}$.

(b) Show that the quotient space X/G is not Hausdorff.

(c) Compute $\pi_1(X/G)$.

Problem 2. (Variant of Hatcher #2.1.3 and Example 2.4) Construct a Δ-complex structure on $\mathbb{R}P^3$ and compute its simplicial homology $H_*^\Delta(\mathbb{R}P^3)$ in all degrees.

Problem 3. For any space X, show that the 0^{th} singular homology $H_0(X)$ is isomorphic to $\mathbb{Z}\pi_0(X)$, the free abelian group on the set of path components of X. Moreover, show that this isomorphism is *natural* in X.

Problem 4. In this problem, we consider the chain complexes of abelian groups C_* and D_* concentrated in degrees 0 and 1 defined as follows. C_* is given by:

$$\cdots \longrightarrow 0 \longrightarrow \mathbb{Z} \xrightarrow{n} \mathbb{Z} \longrightarrow 0 \longrightarrow \cdots$$

while D_* is given by:

$$\cdots \longrightarrow 0 \longrightarrow \mathbb{Z} \twoheadrightarrow \mathbb{Z}/n \longrightarrow 0 \longrightarrow \cdots$$

where $n \geq 2$ is an integer, and $\mathbb{Z} \twoheadrightarrow \mathbb{Z}/n$ denotes the canonical quotient map.

(a) Show that every chain map $f\colon C_* \to D_*$ induces the zero map on homology (in all degrees).

(b) Find a chain map $f\colon C_* \to D_*$ which is *not* chain-homotopic to the zero map. Justify your answer.