Math 9052B/4152B - Algebraic Topology
Winter 2015
Homework 1

Martin Frankland

Due date: Wednesday January 21

Problem 1.

(a) Let $\{X_i\}_{i \in I}$ be a family of spaces and $p_i \colon \prod_i X_i \to X_i$ the projection from the product onto the i^{th} factor. Show that a function $f \colon W \to \prod_i X_i$ is continuous if and only if each of its component functions $p_i f \colon W \to X_i$ is continuous.

(b) Let $\pi \colon X \to Q$ be a quotient map. Show that a function $f \colon Q \to Y$ is continuous if and only if the composite $f\pi \colon X \to Y$ is continuous.

Problem 2. Let X be a connected space such that every point $x \in X$ has a path-connected neighborhood. Show that X is path-connected.

Problem 3. (Variant of Hatcher #0.10) A map $f \colon X \to Y$ is called **null-homotopic** if it is homotopic to a constant map. Show that the following conditions on a space X are equivalent.

1. X is contractible.
2. The identity map $\mathrm{id}_X \colon X \to X$ is null-homotopic.
3. For every space Y, every map $f \colon X \to Y$ is null-homotopic.
4. For every space W, every map $f \colon W \to X$ is null-homotopic.

Problem 4. (Hatcher #0.2) For $n \geq 1$, construct an explicit (strong) deformation retraction of $\mathbb{R}^n \setminus \{0\}$ onto the unit sphere S^{n-1}. (Check that your construction satisfies the required properties.)

Problem 5. (Variant of Hatcher #1.1.10) Let X and Y be spaces, and let $\gamma\colon I \to X \times Y$ be a path from (x_0, y_0) to (x_1, y_1). Construct a path homotopy from γ to a concatenation of paths $\alpha \cdot \beta$ where α is a path in $X \times \{y_0\}$ from (x_0, y_0) to (x_1, y_0) and β is a path in $\{x_1\} \times Y$ from (x_1, y_0) to (x_1, y_1).