Math 535 - General Topology
Fall 2012
Homework 9, Lecture 10/24

Problem 3. Consider the "infinite ladder" $X \subset \mathbb{R}^2$ consisting of two vertical "sides" $\{0\} \times \mathbb{R}$ and $\{1\} \times \mathbb{R}$ along with horizontal "rungs" $[0, 1] \times \{\frac{1}{n}\}$ for all $n \in \mathbb{N}$ as well as $[0, 1] \times \{0\}$. In other words, X is the union

$$X = (\{0, 1\} \times \mathbb{R}) \cup \left([0, 1] \times \left(\{\frac{1}{n} \mid n \in \mathbb{N}\} \cup \{0\} \right) \right) \subset \mathbb{R}^2.$$

Show that X is path-connected, but **not** locally path-connected.

Problem 4. Let (X, d) be a metric space. Given points $x, y \in X$ and $\epsilon > 0$, an ϵ**-chain** from x to y in X is a finite sequence of points

$$x = z_0, z_1, \ldots, z_{n-1}, z_n = y$$

in X such that the distance from one to the next is less than ϵ, i.e. $d(z_{i-1}, z_i) < \epsilon$ for $1 \leq i \leq n$.

Show that if X is connected, then for all $\epsilon > 0$, any two points $x, y \in X$ can be connected by an ϵ-chain.