Math 535 - General Topology
Fall 2012
Homework 8, Lecture 10/19

Recall that a **totally ordered set** is a partially ordered set $(X, \leq)$ where any two elements are comparable: for all $x, y \in X$, either $x \leq y$ or $y \leq x$.

Problem 5. Let $(X, \leq)$ be a totally ordered set. The **order topology** on X is the topology generated by "open rays"

$$(a, \infty) := \{x \in X \mid x > a\}$$
$$(-\infty, a) := \{x \in X \mid x < a\}$$

for any $a \in X$.

a. Show that the order topology on a totally ordered set is always T_1.

b. Show that the order topology on $\mathbb{R}$ with its usual order $\leq$ is the standard (metric) topology on $\mathbb{R}$.

c. An **interval** in a partially ordered set $(X, \leq)$ is a subset $I \subseteq X$ such that for all $x, y \in I$, the condition $x \leq z \leq y$ implies $z \in I$.

Let $(X, \leq)$ be a totally ordered set endowed with the order topology. Show that every connected subspace $A \subseteq X$ is an interval in X.

d. Find an example of totally ordered set $(X, \leq)$, endowed with the order topology, and an interval $A \subseteq X$ which is not a connected subspace.

Problem 6. (Munkres Exercise 23.5) A space is **totally disconnected** if its only connected subspaces are singletons $\{x\}$.

a. Show that every discrete space is totally disconnected.

b. Find an example of totally disconnected space which is not discrete.