Problem 3. Let $f\colon X \to Y$ be a continuous map between topological spaces. Show that f is closed if and only if for all $y \in Y$ and any open subset $U \subseteq X$ satisfying $f^{-1}(y) \subseteq U$, there is an open neighborhood V of y satisfying $f^{-1}(V) \subseteq U$.

Problem 4. Let $f\colon X \to Y$ be a continuous map between topological spaces.

a. Assume that $f\colon X \to Y$ is proper. Let $V \subseteq Y$ be an open subset. Show that the restriction $f|_{f^{-1}(V)}\colon f^{-1}(V) \to V$ is proper.

b. Assume that Y is Hausdorff, and that for all $y \in Y$, there is an open neighborhood V of y such that the restriction $f|_{f^{-1}(V)}\colon f^{-1}(V) \to V$ is proper. Show that $f\colon X \to Y$ is proper.