Math 535 - General Topology
Fall 2012
Homework 8, Lecture 10/15

Problem 1. (Willard Exercise 19B.1) Show that the one-point compactification of $\mathbb{R}^n$ is homeomorphic to the n-dimensional sphere S^n.

Problem 2. (Munkres Exercise 38.4) Let X be any topological space and $f\colon X \hookrightarrow Y$ a compactification of X, where moreover Y is Hausdorff. Show that there is a unique continuous closed surjective map $g\colon \beta X \to Y$ that extends f, meaning $g \circ e = f$, where $e\colon X \to \beta X$ is the canonical evaluation map:

$$\begin{array}{ccc} X & \xrightarrow{e} & \beta X \\ & \searrow^{f} & \downarrow{\scriptstyle g} \\ & & Y. \end{array}$$

Hint: You do not need anything about a specific construction of βX, only its universal property.

Remark. If X admits a Hausdorff compactification, we know that X must be Tychonoff (a.k.a. $T_{3\frac{1}{2}}$), so that $e\colon X \to \beta X$ is in fact an embedding and βX is therefore a compactification of X.

Problem 2 shows in particular that any Hausdorff compactification of X is a quotient of the Stone-Čech compactification βX.