Math 535 - General Topology
Fall 2012
Homework 3, Lecture 9/12

Problem 5. (Bredon Exercise I.3.1) (Munkres Exercise 2.17.6) Let X be a topological space.

a. Let A and B be subsets of X. Show the equality $\overline{A \cup B} = \overline{A} \cup \overline{B}$.

b. Let $\{A_\alpha\}$ be a family of subsets of X. Show the inclusion $\bigcup_\alpha \overline{A_\alpha} \subseteq \overline{\bigcup_\alpha A_\alpha}$.

c. Find an example where the inclusion in part (b) is strict, and X is a *metric* space.

Problem 6. Let X be a metric space and $A \subseteq X$ a subset. The **distance** from a point $x \in X$ to the subset A is

$$d(x, A) := \inf_{a \in A} d(x, a).$$

Show the equivalence $x \in \overline{A}$ if and only if $d(x, A) = 0$.

Problem 7. (Munkres Exercise 2.17.13) The **diagonal** of a space X is the set

$$\Delta := \{(x, x) \mid x \in X\} \subseteq X \times X.$$

Show that X is Hausdorff if and only if the diagonal Δ is closed in $X \times X$.