**Math 535 - General Topology**
**Fall 2012**
**Homework 2, Lecture 9/5**

**Problem 1.** (Brown Exercise 2.4.5) Consider $X = [0, 2] \setminus \{1\}$ as a subspace of the real line $\mathbb{R}$. Show that the subset $[0, 1) \subset X$ is both open and closed in $X$.

**Problem 2.** (Bredon Exercise I.3.8) Let $X$ be a topological space that can be written as a union $X = A \cup B$ where $A$ and $B$ are *closed* subsets of $X$. Let $f \colon X \to Y$ be a function, where $Y$ is any topological space. Assume that the restrictions of $f$ to $A$ and to $B$ are both continuous. Show that $f$ is continuous.

**Problem 3.** A map between topological spaces $f \colon X \to Y$ is called an **open** map if for every open subset $U \subseteq X$, its image $f(U) \subseteq Y$ is open in $Y$.

**a.** (Munkres Exercise 2.16.4) Let $X$ and $Y$ be topological spaces. Show that the projection maps $p_X \colon X \times Y \to X$ and $p_Y \colon X \times Y \to Y$ are open maps.

**b.** Find an example of *metric* spaces $X$ and $Y$, and a closed subset $C \subseteq X \times Y$ such that the projection $p_X(C) \subseteq X$ is *not* closed in $X$.

In other words, the projection maps are (usually) not closed maps.