Math 535 - General Topology
Fall 2012
Homework 14, Lecture 12/7

Problem 5. Let X and Y be topological spaces, where Y is Hausdorff.

a. Consider the set Y^X of *all* functions from X to Y, endowed with the topology of pointwise convergence. Recall that via the correspondence $Y^X \cong \prod_{x \in X} Y$, this corresponds to the product topology.

Show that a collection of functions $\mathcal{F} \subseteq Y^X$ is compact if and only if the following two conditions hold:

1. $\mathcal{F}$ is closed in Y^X;
2. For all $x \in X$, the projection $p_x(\mathcal{F}) = \{f(x) \mid f \in \mathcal{F}\} \subseteq Y$ has compact closure in Y.

b. Let $C(X, Y)$ be endowed with the compact-open topology, and let $\mathcal{F} \subseteq C(X, Y)$ be a compact subspace. Show that $\mathcal{F}$ satisfies the conditions 1. and 2. listed in part (a), i.e.

1. $\mathcal{F}$ viewed as a subset of Y^X is closed **with respect to the topology of pointwise convergence**;
2. For all $x \in X$, the projection $p_x(\mathcal{F}) = \{f(x) \mid f \in \mathcal{F}\} \subseteq Y$ has compact closure in Y.

Problem 6. (Munkres Exercise 47.1) For each of the following subsets $\mathcal{F} \subset C(\mathbb{R}, \mathbb{R})$, say if $\mathcal{F}$ is equicontinuous of not, and prove your answer.

a. $\mathcal{F} = \{f_n \mid n \in \mathbb{N}\}$ where $f_n(x) = x + \sin nx$.

b. $\mathcal{F} = \{g_n \mid n \in \mathbb{N}\}$ where $g_n(x) = n + \sin x$.

c. $\mathcal{F} = \{h_n \mid n \in \mathbb{N}\}$ where $h_n(x) = |x|^{\frac{1}{n}}$.

d. $\mathcal{F} = \{k_n \mid n \in \mathbb{N}\}$ where $k_n(x) = n \sin\left(\frac{x}{n}\right)$.