Math 535 - General Topology
Fall 2012
Homework 10, Lecture 10/29

Problem 1. Let **Top** denote the category of topological spaces and continuous maps, and let **CHaus** denote the category of compact Hausdorff topological spaces and continuous maps. Show that the Stone-Čech construction

$$\beta\colon \mathbf{Top} \to \mathbf{CHaus}$$

is a functor.

Note: So far we know that β sends objects of **Top** to objects of **CHaus**. There remain three things to check.

Problem 2. Let $\mathbf{Top}_*$ denote the category of pointed topological spaces and pointed continuous maps. To a space X, one can associate the pointed space

$$X_+ := X \amalg \{*\}$$

(with the coproduct topology) called "X with a disjoint basepoint", where $* \in X_+$ is the basepoint. To a continuous map $f\colon X \to Y$, one can assign the pointed continuous map

$$f_+\colon (X_+, *) \to (Y_+, *)$$

defined by

$$\begin{cases} f_+(x) = f(x) & \text{if } x \in X \\ f_+(*) = *. \end{cases}$$

One readily checks that this assignment makes the disjoint basepoint construction

$$(-)_+\colon \mathbf{Top} \to \mathbf{Top}_*$$

into a functor.

a. Show that for any space X and pointed space (Y, y_0), there is a bijection

$$\mathrm{Hom}_{\mathbf{Top}_*}\left((X_+, *), (Y, y_0)\right) \cong \mathrm{Hom}_{\mathbf{Top}}\left(X, Y\right).$$

b. Show that the bijection in part (a) induces a bijection

$$[(X_+, *), (Y, y_0)]_* \cong [X, Y]$$

where $[(A, a_0), (B, b_0)]_* := \mathrm{Hom}_{h\mathbf{Top}_*}\left((A, a_0), (B, b_0)\right)$ denotes the set of pointed homotopy classes of pointed continuous maps from (A, a_0) to (B, b_0).

As usual, $[X, Y] := \mathrm{Hom}_{h\mathbf{Top}}(X, Y)$ denotes the set of homotopy classes of continuous maps from X to Y.