Math 527 - Homotopy Theory
Spring 2013
Homework 6, Lecture 2/22

Problem 3. Let X be a CW-complex with skeletal filtration $X_0 \subseteq X_1 \subseteq \ldots \subseteq X = \operatorname{colim}_n X_n$. Show that for any $k \geq 0$, there is a natural isomorphism

$$\pi_k(X) \cong \operatorname*{colim}_n \pi_k(X_n).$$

Remark. The same statement holds for homology $H_k(X) \cong \operatorname{colim}_n H_k(X_n)$, essentially for the same reason.

Problem 4. In this problem, feel free to refer to Homework 5 Problem 4. Consider infinite-dimensional real projective space $\mathbb{R}P^\infty = \operatorname{colim}_n \mathbb{R}P^n$.

a. Compute all homotopy groups of $\mathbb{R}P^\infty$.

b. Show that $\mathbb{R}P^2$ and $S^2 \times \mathbb{R}P^\infty$ are not homotopy equivalent.