

Math 527 - Homotopy Theory
Spring 2013
Homework 3, Lecture 1/28

Problem 1. An **H-space** (named after Hopf) is a pointed space (X, e) equipped with a “multiplication” map $\mu: X \times X \rightarrow X$ such that the basepoint e is a two-sided unit up to pointed homotopy. In other words, both maps

$$\mu(e, -): X \rightarrow X$$

$$\mu(-, e): X \rightarrow X$$

and pointed-homotopic to the identity map id_X . Note that μ is not assumed to be associative, not even up to homotopy.

Show that the fundamental group $\pi_1(X, e)$ of an H-space is abelian.