

**Math 527 - Homotopy Theory**  
**Spring 2013**  
**Homework 1, Lecture 1/14**

**Problem 1.** Show that the following conditions on a topological space  $X$  are equivalent.

1.  $X$  is contractible.
2. The identity map  $\text{id}_X: X \rightarrow X$  is null-homotopic.
3. For any space  $Y$ , every continuous map  $f: X \rightarrow Y$  is null-homotopic.
4. For any space  $W$ , every continuous map  $f: W \rightarrow X$  is null-homotopic.
5. For any space  $W$ , the projection map  $p_W: W \times X \rightarrow W$  is a homotopy equivalence.