Math 527 - Homotopy Theory Spring 2013 Homework 11, Lecture 4/5

Problem 3. Let X be a CW complex, with n-skeleton X_n , and let Y be a path-connected simple space. Let $n \ge 2$, and let $f_n, g_n: X_n \to Y$ be two maps which agree on X_{n-1} , i.e.

$$f_n|_{X_{n-1}} = g_n|_{X_{n-1}}.$$

Let $d(f_n, g_n) \in C^n(X; \pi_n Y)$ denote their difference cochain.

Show that $f_n \simeq g_n$ rel X_{n-2} holds if and only if $[d(f_n, g_n)] = 0 \in H^n(X; \pi_n Y)$ holds, i.e. $d(f_n, g_n)$ is a coboundary.