Math 527 - Homotopy Theory
Spring 2013
Homework 10, Lecture 3/29

Problem 3. Let $n \geq 0$ and let X be a space with the homotopy type of a CW complex. Consider the Postnikov truncation map $t_n \colon X \to P_n X$, which may be assumed a relative CW complex.

Let $f \colon X \to Z$ be any map, where Z is an Eilenberg-MacLane space of type (G, k) for some abelian group G and $k \leq n$.

Show that there exists a map $g \colon P_n X \to Z$ satisfying $f \simeq g \circ t_n$, and this map g is **unique up to homotopy**. Here, g makes the diagram

$$\begin{array}{ccc} X & \xrightarrow{t_n} & P_n X \\ & \underset{f}{\searrow} & \Big\downarrow{\scriptstyle g} \\ & & Z \end{array}$$

commute up to homotopy.

Remark. The statement still holds when Z is a product of such Eilenberg-MacLane spaces. However, if Z is more complicated, but still n-truncated (i.e. $\pi_i(Z) = 0$ for $i > n$), then such a factorization $g \colon P_n X \to Z$ still exists, but its homotopy class **need not** be unique.