Math 527 - Homotopy Theory
Spring 2013
Homework 10, Lecture 3/25

Problem 1. Let $n \geq 2$. Consider the wedge $X = S^1 \vee S^n$.

a. Show that the n^{th} homotopy group of X is a free $\pi_1(X)$-module on one generator:

$$\pi_n(X) \cong \mathbb{Z}[\pi_1(X)] \cong \mathbb{Z}[t, t^{-1}].$$

b. Take a representative $f \colon S^n \to X$ of the class $2t - 1 \in \pi_n(X)$ and form a space Y by attaching an $(n+1)$-cell to X via f, as illustrated in the cofiber sequence:

$$S^n \xrightarrow{f} X \xrightarrow{j} Y.$$

Show that the composite $S^1 \xrightarrow{\iota_1} X \xrightarrow{j} Y$ induces an isomorphism on integral homology:

$$H_*(S^1; \mathbb{Z}) \xrightarrow{\simeq} H_*(Y; \mathbb{Z}).$$

Here $\iota_1 \colon S^1 \hookrightarrow X$ is the wedge summand inclusion.

c. Show that the same map $j \circ \iota_1 \colon S^1 \to Y$ induces an isomorphism on π_k for $k < n$ but *not* on π_n.