Math 416 - Abstract Linear Algebra
Fall 2011, section E1
Preview of Gauss-Jordan elimination (§ 2.2)

Consider the linear system

$$\begin{cases} x_1 - x_2 & = 3 \\ 4x_1 + 3x_2 & = 5 \end{cases}$$

whose augmented matrix is

$$\left[\begin{array}{cc|c} 1 & -1 & 3 \\ 4 & 3 & 5 \end{array}\right].$$

We solve the system using Gauss-Jordan elimination:

$$\left[\begin{array}{cc|c} 1 & -1 & 3 \\ 4 & 3 & 5 \end{array}\right] \overset{R_2 - 4R_1}{\sim} \left[\begin{array}{cc|c} 1 & -1 & 3 \\ 0 & 7 & -7 \end{array}\right]$$

$$\overset{\frac{1}{7}R_2}{\sim} \left[\begin{array}{cc|c} 1 & -1 & 3 \\ 0 & 1 & -1 \end{array}\right]$$

$$\overset{R_1 + R_2}{\sim} \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -1 \end{array}\right].$$

The unique solution is $\overrightarrow{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$.

Remark 1: The reduced row echelon form of the augmented matrix (last step) is saying

$$x_1 = 2$$

$$x_2 = -1$$

whence the conclusion.

Remark 2: Let us check that $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ is indeed a solution, in all three points of view on linear systems. As a system of linear equations:

$$\begin{cases} (2) - (-1) & = 3 \\ 4(2) + 3(-1) & = 5. \end{cases}$$

As a matrix-vector equation:

$$\begin{bmatrix} 1 & -1 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}.$$

As a vector equation:

$$2 \begin{bmatrix} 1 \\ 4 \end{bmatrix} - 1 \begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 8 \end{bmatrix} + \begin{bmatrix} 1 \\ -3 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}.$$