Math 285 - Intro Differential Equations
Spring 2011, sections G1 and X1
Practice Midterm 1, Wednesday February 23

Name: ____

Section (circle): X1 at 12pm G1 at 3pm

In the actual exam, no calculators, electronic devices, books, or notes may be used.
Show your work. No credit for answers without justification.
You may omit units for ease of reading, except in final answers which require units.
Good luck!

1. __________/10
2. __________/10
3. __________/10
4. __________/10
5. __________/10

Total: __________/50

Problem 1a. (4 pts) Find the general solution of the equation $yy' = (x-5)^3$.

b. (3 pts) Find a particular solution that is defined on the real line, i.e. for all values of x.

c. (3 pts) Find a particular solution that is defined on some interval but cannot be extended to the real line.

Problem 2. (10 pts) Find the general solution of the equation $3xy^2y' = 3x^4 + y^3$.

Problem 3. A population of gazelles has a constant death rate $\delta = 0.01$ (deaths per gazelle per month) and a birth rate (per gazelle per month) $\beta = 0.03 - \beta_1 P$ which decreases linearly with the population size P, because of limited resources. Initially, the population numbers 20 and has a rate of change of 0.32 gazelle per month.

a. (5 pts) Find $P(t)$, the population size as a function of time.

b. (3 pts) Does the population ever reach 50? When?

c. (2 pts) What happens to the population in the long run? If it tends to a constant number, **find that number**; if it explodes to infinity, **find when** that happens.

Problem 4. A boat is cruising at speed v_0 when the engine is turned off (at time $t = 0$). Assume that the boat encounters resistance proportional to $v^{\frac{3}{2}}$, so that its velocity v satisfies the equation $v' = -av^{\frac{3}{2}}$ for some constant a.

a. (5 pts) Find $v(t)$, the velocity as a function of time.

b. (3 pts) Find $x(t)$, the position of the boat (relative to where it was when the engine was turned off).

c. (2 pts) In the long run, would the boat coast infinitely far, or only a finite distance? If the latter, find that distance.

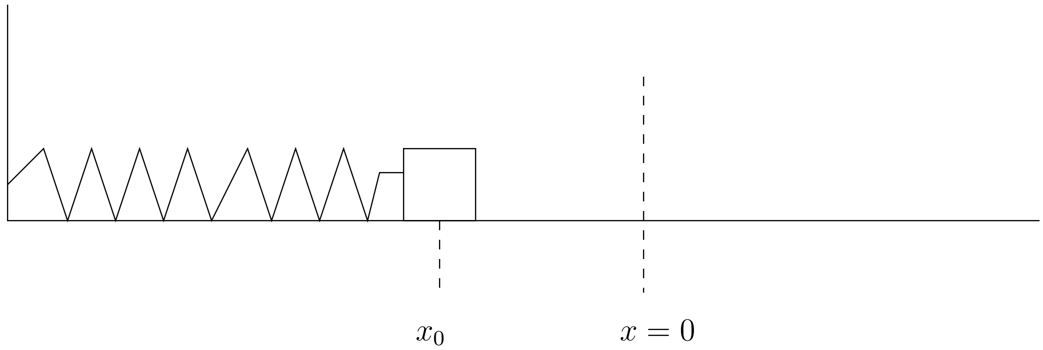

Figure 1: Mass attached to a spring.

Problem 5. A mass of 1 kg is attached to a spring and moves horizontally on a surface (figure 1). Let $x(t)$ be the position of the mass at time t, relative to the equilibrium position $x = 0$.

By Hooke's law, the spring exerts a force of $-kx$, where the spring constant is $k = 1\,\mathrm{N/m}$.

The mass is placed at initial position x_0 and released from there.

a. (3 pts) Assume there is no friction on the surface. Find $x(t)$.

b. (4 pts) Assume there is friction against the surface, proportional to velocity, so that the friction force is $-cx'$. If there is too much friction, then the mass will move towards the middle, slow down, and never come back.

Assume the friction coefficient is $c = 10\,\mathrm{N}/(\mathrm{m}/\mathrm{s})$. Find $x(t)$.

c. (3 pts) What is the specific value of the friction coefficient c above which friction prevents the mass from oscillating?