Tuesday, January 31 ** *Worksheet 5 - Visualizing quadric surfaces.*

1. **Elliptic paraboloid.** Equation
$$z = Ax^2 + By^2$$
where A and B have the **same** sign.
   (a) What happens if either A or B is 0? What if they both are? Should any of these objects be called "elliptic" paraboloids?
   (b) What would happen if the sliders included negative values for A and B?

2. **Hyperbolic paraboloid.** Equation
$$z = Ax^2 + By^2$$
where A and B have **different** signs.
   (a) What does the horizontal cross section given by $z = 0$ look like? Check on the first picture, and also look at the equation when $z = 0$. Is this still a hyperbola?
   (b) How would $z = y^2 - x^2$ look different from $z = x^2 - y^2$?

3. **Ellipsoid.** Equation
$$\left(\frac{x}{A}\right)^2 + \left(\frac{y}{B}\right)^2 + \left(\frac{z}{C}\right)^2 = 1.$$
   (a) What needs to happen for an ellipsoid to be a sphere?
   (b) The sliders don't actually go all the way to 0. Make the values as small as you can and zoom in to verify this; you'll find you have a very small sphere. (Its radius is 0.1, as it happens.) Why shouldn't the sliders go all the way to 0?

4. **Double cone.** Equation
$$z^2 = Ax^2 + By^2.$$
   (a) Why aren't any of the vertical or horizontal cross sections parabolas?
   (b) Explain what happens when either $A = 0$ or $B = 0$. Why don't you get a cone?
   (c) Similarly, what are the cross sections given by $x = 0$ or $y = 0$? Are these hyperbolas?

5. **Hyperboloid of one sheet.** Equation
$$\left(\frac{x}{A}\right)^2 + \left(\frac{y}{B}\right)^2 - \left(\frac{z}{C}\right)^2 = 1.$$
   (a) Once again, the sliders don't go all the way to 0. Why not? Make all of them as small as possible and zoom in to see the resulting hyperboloid.
   (b) Look at the equation. What should happen when $x = A$ or $x = -A$? Check this in the first picture; recall that $A = 1$ there.

(c) Does there always have to be a "hole" through the hyperboloid, or could the sides touch at the origin? In other words, could the cross section given by $z = 0$ ever be a point instead of an ellipse? Experiment with the second picture; be sure to look directly from the top and zoom in before just assuming that the hole is gone.

6. **Hyperboloid of two sheets.** Equation

$$-\left(\frac{x}{A}\right)^2 - \left(\frac{y}{B}\right)^2 + \left(\frac{z}{C}\right)^2 = 1.$$

(a) Go back to the equation and figure out why larger values of A and B make the hyperboloid flatter, not steeper.

(b) Does there always need to be a gap between the two sheets, or could they touch?