Tuesday, April 24 ∗∗ *Worksheet 22 - Surface integrals of vector fields and related theorems*

1. Consider the region D in $\mathbb{R}^3$ bounded by the xy-plane and the surface $x^2 + y^2 + z = 1$.
   (a) Make a sketch of D.
   (b) The boundary of D, denoted ∂D, has two parts: the curved top S_1 and the flat bottom S_2. Parameterize S_1 and calculate the flux of $\mathbf{F} = (0, 0, z)$ through S_1 with respect to the upward pointing unit normal vector field. Check you answer with the instructor.
   (c) Without doing the full calculation, determine the flux of $\mathbf{F}$ through S_2 with the downward pointing normals.
   (d) Determine the flux of $\mathbf{F}$ through ∂D with the outward pointing normals.
   (e) Apply the Divergence Theorem and your answer in (d) to find the volume of D. Check your answer with the instructor.

2. Consider the vector field $\mathbf{F} = (-y, x, z)$.
   (a) Compute curl $\mathbf{F}$.
   (b) For the surface S_1 above, evaluate $\iint_{S_1} (\text{curl } \mathbf{F}) \cdot \mathbf{n} \, dA$.
   (c) Check your answer in (b) using Stokes' Theorem.

3. If time remains:
   (a) Check your answer in 1(e) by directly calculating the volume of D.
   (b) Repeat 2 (b-c) for the surface S_2 and also for the surface ∂D. What exactly does 2(c) mean for the surface ∂D?
   (c) For the vector field $\mathbf{F} = (-y, x, z)$ from the second problem, compute $\text{div}(\text{curl}\, \mathbf{F})$. Now suppose $\mathbf{F} = (F_1, F_2, F_3)$ is an arbitrary vector field. Can you say anything about the function $\text{div}(\text{curl}\, \mathbf{F})$?