

**Tuesday, April 10** \*\* *Worksheet 20 - Surface Parameterpalooza*

1. Let  $S$  be the portion of the plane  $x + y + z = 1$  which lies in the positive octant.
  - (a) Draw a picture of  $S$ .
  - (b) Find a parametrization  $\mathbf{r}: D \rightarrow S$ , being sure to clearly indicate the domain  $D$ . Check your answer with the instructor.
  - (c) Use your answer in (b) to compute the area of  $S$  via an integral over  $D$ .
  - (d) Check your answer in (c) using only things you learned in the first few weeks of this class.

2. Consider the surface  $S$  which is the part of  $z + x^2 + y^2 = 1$  where  $z \geq 0$ .

- (a) Draw a picture of  $S$ .
- (b) Find a parametrization  $\mathbf{r}: D \rightarrow S$ . Check your answer with the instructor.

3. Let  $S$  be the surface given by the following parametrization. Let  $D = [-1, 1] \times [0, 2\pi]$  and define

$$\mathbf{r}(u, v) = (u \cos v, u \sin v, v).$$

- (a) Consider the vertical line segment  $L = \{u = 0\}$  in  $D$ . Describe geometrically the image of  $L$  under  $\mathbf{r}$ .
- (b) Repeat for the vertical segments where  $u = -1$  and  $u = 1$ .
- (c) Use your answers in (a) and (b) to make a sketch of  $S$ .

4. Consider the ellipsoid  $E$  given by  $\frac{x^2}{9} + \frac{y^2}{4} + z^2 = 1$ .

- (a) Draw a picture of  $E$ .
- (b) Find a parametrization of  $E$ . Hint: Find a transformation  $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$  which takes the unit sphere  $S$  to  $E$ , and combine that with our existing parametrization of the plain sphere  $S$ .