Tuesday, March 6 $\quad **\quad$ *Worksheet 13 - Integrating vector fields.*

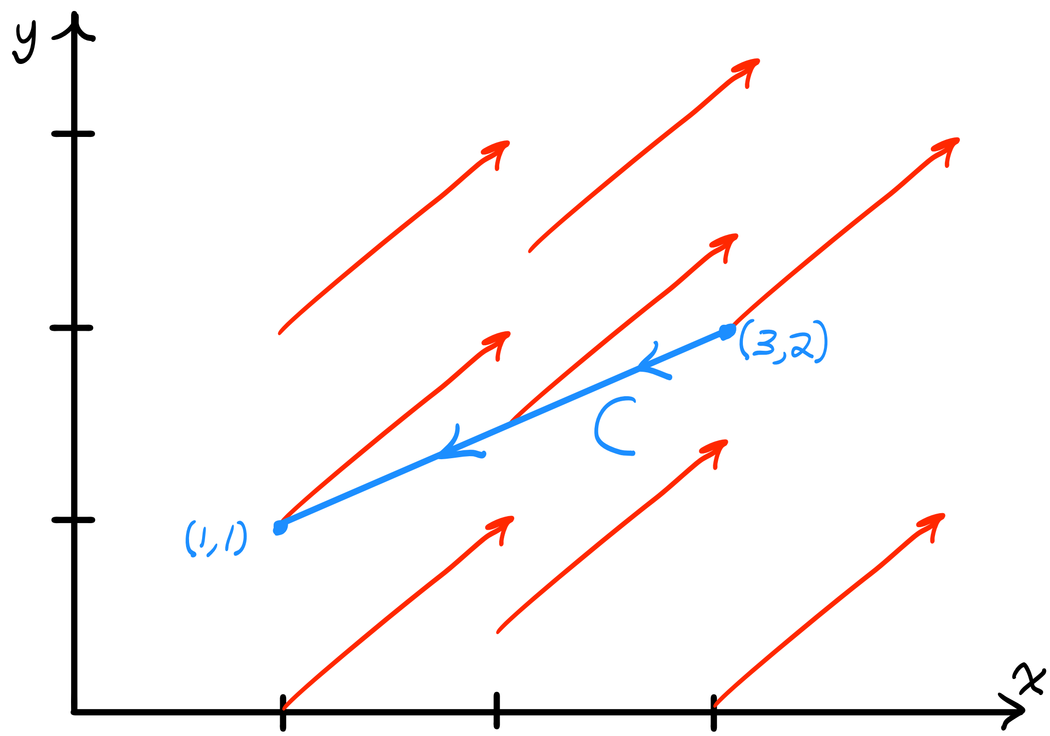
1. Consider the vector field $\mathbf{F} = (y, 0)$ on $\mathbb{R}^2$.

   (a) Draw a sketch of $\mathbf{F}$ on the region where $-2 \leq x \leq 2$ and $-2 \leq y \leq 2$. Check you answer with the instructor.

   (b) Consider the following two curves which *start* at $A = (-2, 0)$ and *end* at $B = (2, 0)$, namely the line segment C_1 and upper semicircle C_2.
   Add these curves to your sketch, and compute both $\int_{C_1} \mathbf{F} \cdot d\mathbf{r}$ and $\int_{C_2} \mathbf{F} \cdot d\mathbf{r}$. Check you answers with the instructor.

   (c) Based on your answer in (b), could $\mathbf{F}$ be ∇f for some $f\colon \mathbb{R}^2 \to \mathbb{R}$? Explain why or why not.

2. Consider the curve C and vector field $\mathbf{F}$ shown below.



   (a) Calculate $\mathbf{F} \cdot \mathbf{T}$, where here $\mathbf{T}$ is the unit tangent vector along C. Without parametrizing C, evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ by using the fact that it is equal to $\int_C \mathbf{F} \cdot \mathbf{T}\, ds$.

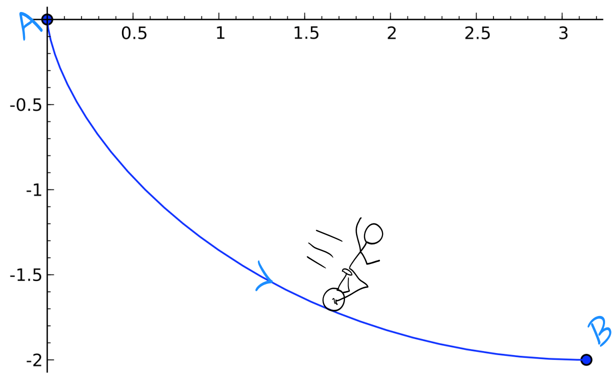
   (b) Find a parametrization of C and a formula for $\mathbf{F}$. Use them to check your answer in (a) by computing $\int_C \mathbf{F} \cdot d\mathbf{r}$ explicitly.

Note: More problems on the back.

3. Consider the points $A = (0, 0)$ and $B = (\pi, -2)$. Suppose an object of mass m moves from A to B and experiences the constant force $\mathbf{F} = -mg\mathbf{j}$, where g is the gravitational constant.

   (a) If the object follows the straight line from A to B, calculate the work W done by gravity using the formula from the first week of class.

   (b) Now suppose the object follows half of an inverted cycloid C as shown below. Explicitly parametrize C and use that to calculate the work done via a line integral.



   (c) Find a function $f\colon \mathbb{R}^2 \to \mathbb{R}$ so that $\nabla f = \mathbf{F}$. Use the Fundamental Theorem of Line Integrals to check your answers for (a) and (b). Have you seen the quantity $-f$ anywhere before? If so, what was its name?

4. If you get this far, work #48 from Section 16.2:

48. Experiments show that a steady current I in a long wire produces a magnetic field $\mathbf{B}$ that is tangent to any circle that lies in the plane perpendicular to the wire and whose center is the axis of the wire (as in the figure). *Ampère's Law* relates the electric current to its magnetic effects and states that

$$\int_C \mathbf{B} \cdot d\mathbf{r} = \mu_0 I$$

where I is the net current that passes through any surface bounded by a closed curve C, and μ_0 is a constant called the permeability of free space. By taking C to be a circle with radius r, show that the magnitude $B = |\mathbf{B}|$ of the magnetic field at a distance r from the center of the wire is

$$B = \frac{\mu_0 I}{2\pi r}$$