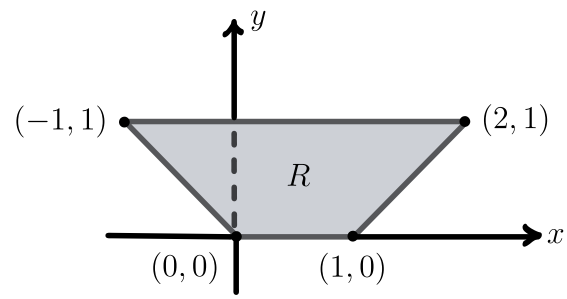
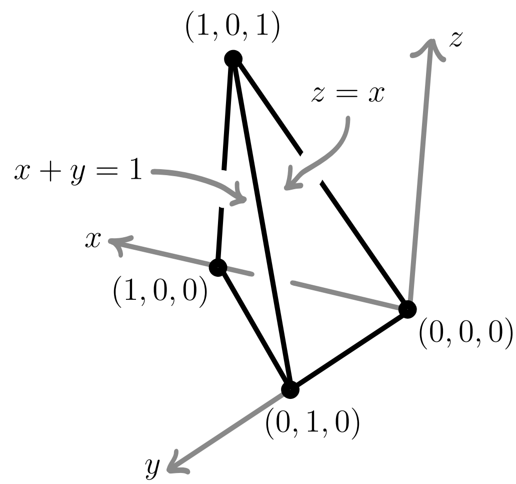
1. Let R be the region of integration pictured below right. Evaluate $\iint_R 6y\ dA$. **(4 points)**



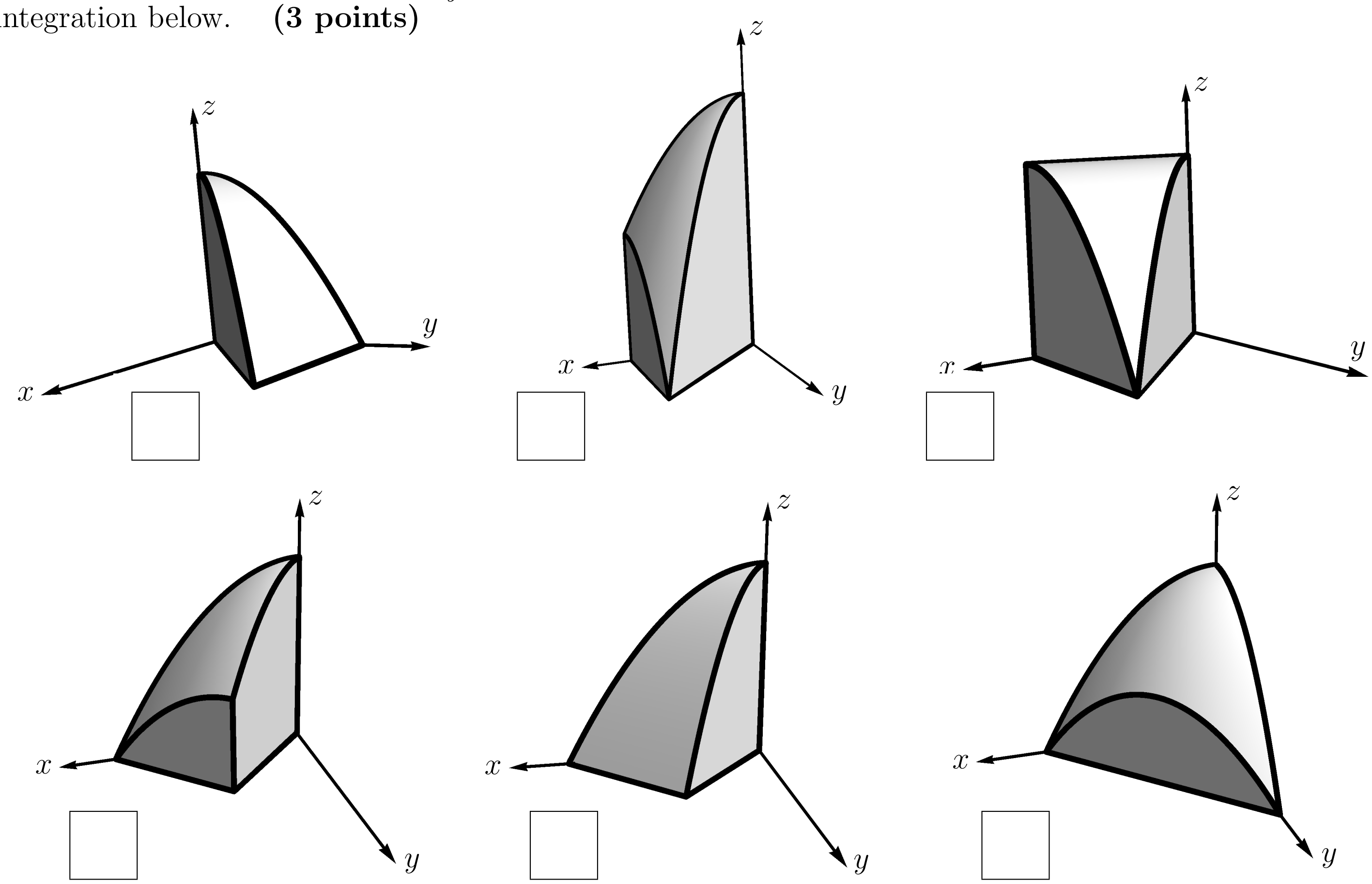
$\iint_R 6y\ dA =$

2. Set up, but DO NOT EVALUATE, a triple integral that computes the volume of the tetrahedron shown at right. **(5 points)**

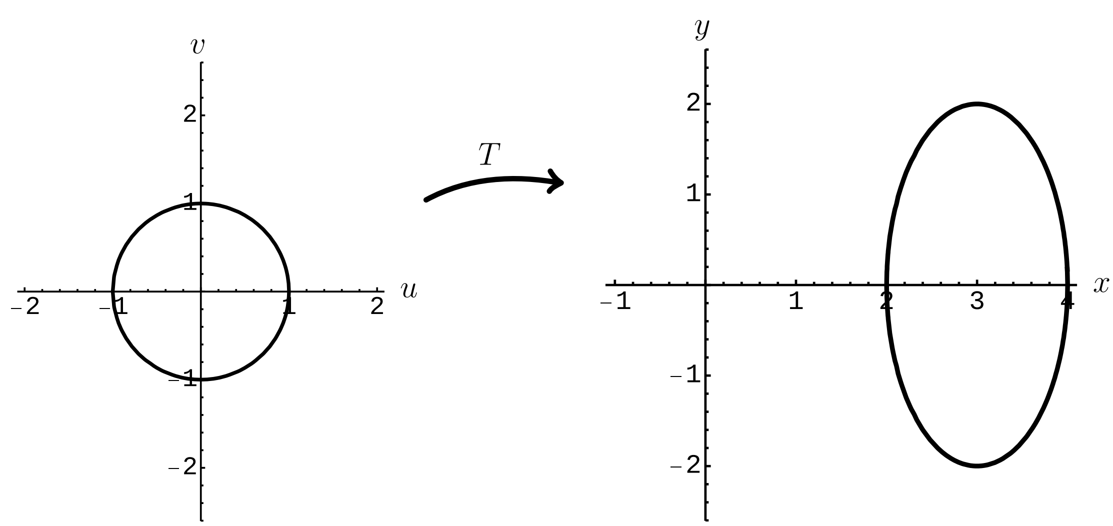


3. Set up, but DO NOT EVALUATE, a triple integral that computes the volume of the region that lies inside the sphere $x^2 + y^2 + z^2 = 2$ and above the cone $z = \sqrt{x^2 + y^2}$. **(5 points)**

4. Consider the triple integral $\displaystyle\int_0^{1/2}\int_y^{1-y}\int_0^{1-x^2-y^2} f(x,y,z)\ dz\ dx\ dy$. Mark the corresponding region of integration below. **(3 points)**



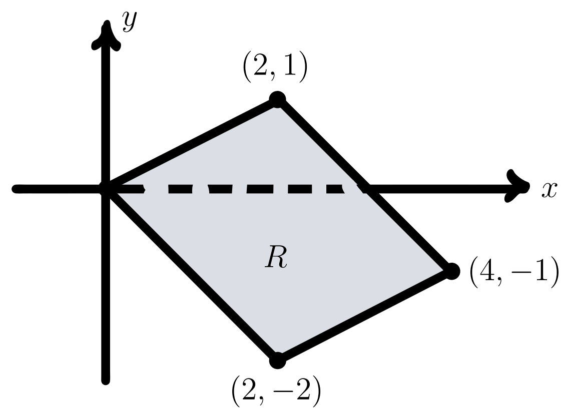
5. Find a transformation $T\colon\ \mathbb{R}^2 \to \mathbb{R}^2$ which takes the unit circle to the ellipse given by $(x-3)^2 + \dfrac{y^2}{4} = 1$ as shown. **(3 points)**



$T(u,v) = \Big(\qquad\qquad , \qquad\qquad \Big)$

6. Let R be the region in the xy-plane depicted below right. Let $T(u,v) = \big(2u+v, u-v\big)$.

(a) Find a rectangle S in the uv-plane whose image under T (that is, the collection of points $T(u,v)$ for all choices of (u,v) in S) is exactly R. **(3 points)**

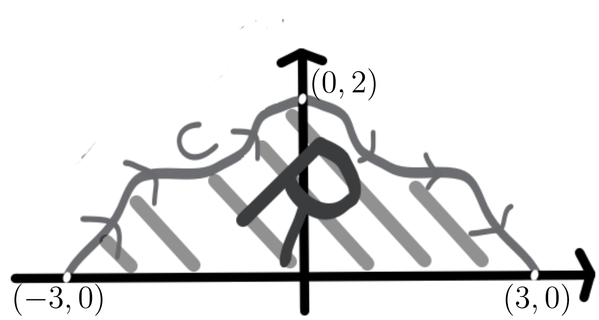


ANSWER: $S = \left\{ (u,v) \;\middle|\; \boxed{} \le u \le \boxed{},\ \boxed{} \le v \le \boxed{} \right\}.$

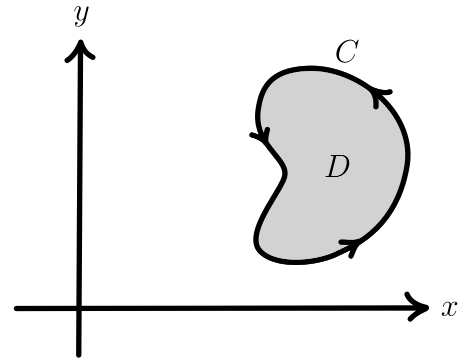
(b) Set up, but DO NOT EVALUATE, the integral $\iint_R \cos(x)\, dA$ as an integral in the (u,v)-coordinates. If you can't do part (a), leave the limits of integration blank. **(5 points)**

$$\iint_R \cos(x)\; dA =$$

7. Let $\mathbf{F}(x,y) = \langle x^2, x^2\cos(y)\rangle$. Then $\iint_R \left[\frac{\partial}{\partial x}\left(x^2\cos(y)\right) - \frac{\partial}{\partial y}\left(x^2\right)\right]\, dA = 0$ where R is the region shown below. Compute $\int_C \mathbf{F}\cdot d\mathbf{r}$, where C is the pictured curve that goes from $(-3,0)$ to $(3,0)$ via $(0,2)$. **(3 points)**



8. Consider the region D in the plane bounded by the curve C as shown at right. For each part, circle the best answer. **(1 point each)**



(a) For $\mathbf{F}(x, y) = \langle x + 1, y^2 \rangle$, the integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ is

negative	zero	positive

(b) The integral $\int_C \big(-y\,dx + 2\,dy \big)$ is

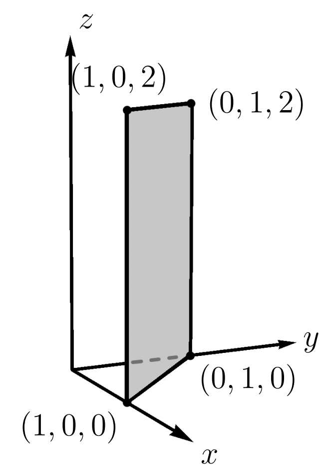
negative	zero	positive

(c) The integral $\iint_D (y - x)\,dA$ is

negative	zero	positive

9. For each surface S below, give a parameterization $\mathbf{r}\colon D \to S$. Be sure to explicitly specify the domain D and call your parameters u and v.

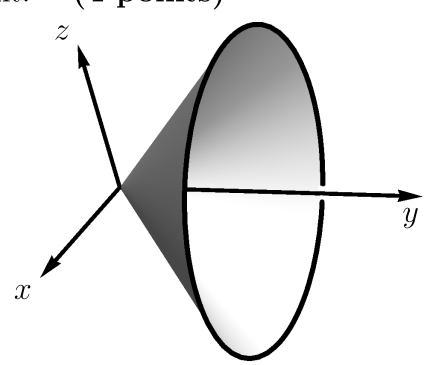
(a) The rectangle in $\mathbb{R}^3$ with vertices $(1, 0, 0), (0, 1, 0), (1, 0, 2), (0, 1, 2)$. **(3 points)**



$D = \Big\{ \quad \le u \le \quad \text{and} \quad \le v \le \quad \Big\}$

$\mathbf{r}(u, v) = \Big\langle \qquad , \qquad , \qquad \Big\rangle$

(b) The portion of cone $y = \sqrt{x^2 + z^2}$ for $0 \le y \le 1$ which is shown at right. **(4 points)**

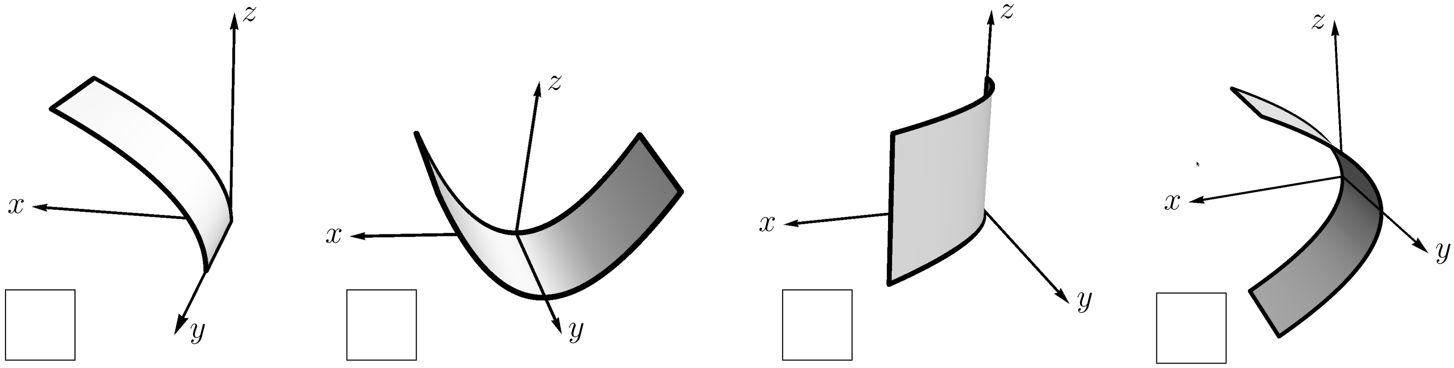


$D = \Big\{ \quad \le u \le \quad \text{and} \quad \le v \le \quad \Big\}$

$\mathbf{r}(u, v) = \Big\langle \qquad , \qquad , \qquad \Big\rangle$

10. Consider the surface S parameterized by $\mathbf{r}(u, v) = (v^2,\ u,\ v)$ for $0 \le u \le 1$ and $0 \le v \le 1$.

(a) Mark the correct picture of S below. **(2 points)**



☐ ☐ ☐ ☐

(b) Evaluate the integral $\iint_S z\ dA$. **(6 points)**

$\iint_S z\ dA\ =$

11. Consider the solid described as follows using cylindrical coordinates: E is the region inside the paraboloid $z = 1 - r^2$ and where $0 \le \theta \le \pi$ and $z \ge 0$. Choose one double integral and one triple integral below that compute the volume of E. **(1 point each)**

☐ $\displaystyle\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \int_0^{\sqrt{1-x^2-y^2}} 1\ dz\ dx\ dy$

☐ $\displaystyle\int_0^1 \int_0^{\sqrt{1-z}} 2\sqrt{1-z-y^2}\ dy\ dz$

☐ $\displaystyle\int_0^1 \int_{-\sqrt{1-z}}^{\sqrt{1-z}} \int_0^{\sqrt{1-z-x^2}} 1\ dy\ dx\ dz$

☐ $\displaystyle\int_0^1 \int_0^{\sqrt{1-z^2}} 2\sqrt{1-y^2-z^2}\ dy\ dz$

☐ $\displaystyle\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \int_0^{1-\sqrt{x^2+y^2}} 1\ dz\ dx\ dy$

☐ $\displaystyle\int_0^1 \int_0^{1-z} 2\sqrt{(1-z)^2-y^2}\, dy\ dz$