Calculus 2502A - Advanced Calculus I

Fall 2014
Practice Midterm

Name (please print): ____

Student ID: ___________________________________

- On the real exam, no calculators, electronic devices, books, or notes may be used.
- Show your work. No credit for answers without justification.
- Good luck!

(1) __________/10

(2) __________/10

(3) __________/10

(4) __________/10

(5) __________/10

(6) __________/10

(7) __________/10

(8) __________/10

(9) __________/10

(10) __________/10

(11) __________/10

(12) __________/10

(13) __________/25

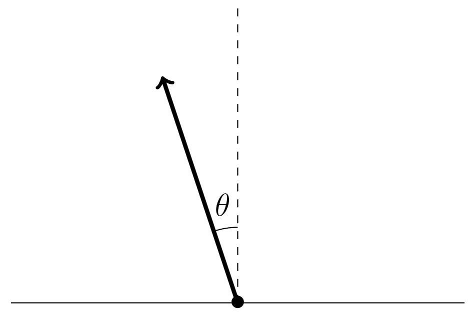
Total: __________/145

List of formulas.

Curvature: $\kappa(t) = |\frac{d\vec{T}}{ds}|$, where $\vec{T}$ = unit tangent vector, and s = arc length.

Newton's Law: $\vec{F} = m\vec{a}$, where $\vec{F}$ = force, m = mass, and $\vec{a}$ = acceleration.

Problem 1. (10 points) A boat wants to head due north. The water current is going northeast, with a speed of 5 km/h. The boat moves at full speed, which is 50 km/h (relative to the water). In what direction should the boat head (relative to the water) so that its actual motion is due north?
Give your answer as an angle θ of deviation from the north direction, going counterclockwise. For example, $\theta = 0$ would be north and $\theta = \frac{\pi}{4}$ would be northwest.



Problem 2. (10 points) Find the distance from the point $P = (5, 2, 1)$ to the plane defined by the equation $2x + y - 4z = 3$.

Problem 3. (10 points) Find an equation for the plane that contains both the point $P = (3, 1, 5)$ and the line with parametric equations:

$$\begin{cases} x(t) = 4 - t \\ y(t) = 2t \\ z(t) = -2 + t \qquad \text{for } t \in \mathbb{R}. \end{cases}$$

Problem 4. (10 points) Find a vector equation for the line of intersection of the two planes P_1 and P_2 defined by the equations:

$$P_1 : 3x + y - 2z = 4$$
$$P_2 : x + 2y + z = 1.$$

Problem 5. (10 points) Show that the equation

$$4x^2 + 36y^2 - 72y + 9z^2 + 36z + 9 = 0$$

in $\mathbb{R}^3$ defines an ellipsoid, and find its center.

Problem 6. (10 points) Let P be the point $(-3, -2, -1)$ in rectangular coordinates.

a) Find the cylindrical coordinates of P.

b) Find the spherical coordinates of P.

Problem 7. (10 points) Find all points of intersection (if any) of the line through the points $(1, 1, 2)$ and $(3, 3, 0)$ and the surface whose equation in spherical coordinates is $\rho = 6 \cos \phi$.

Problem 8. (10 points) Let C be a curve parametrized by a vector function $\vec{r} : \mathbb{R} \to \mathbb{R}^3$. Assume that for all $t \in \mathbb{R}$, the position vector $\vec{r}(t)$ is orthogonal to the tangent vector $\vec{r}\,'(t)$. Show that the curve C lies entirely on a sphere centered at the origin. (Assume that $\vec{r}$ is differentiable for all $t \in \mathbb{R}$.)

Problem 9. (10 points) Let C be the curve of intersection of the parabolic cylinder $x^2 = 2y$ and the surface $3z = xy$. Find the length of C from the origin to the point $(6, 18, 36)$.

Problem 10. (10 points) Find the curvature of the curve parametrized by $\vec{r}(t) = (t^2, 7, t)$ at the point $(1, 7, 1)$.

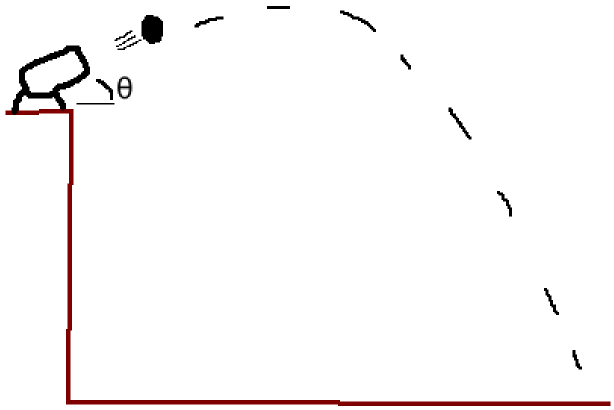


FIGURE 1

Problem 11. (10 points) A cannonball of mass 7 kg is shot from a cliff located 100 meters above ground, with an angle of $\theta = 30°$ above the horizontal, at an initial speed of 10 m/s. Find the **speed** of the ball when it hits the ground. (Recall that speed is the magnitude of the velocity.)

In this problem, ignore air resistance, and write the gravitational acceleration on Earth as g, which is approximately $g = 9.8\,\mathrm{N/kg} = 9.8\,\mathrm{m/s^2}$.

(More space.)

Problem 12. (10 points) A particle is moving in space according to the position function

$$\vec{r}\,(t) = \left(t^2 + 3, 4t - 5, 3t + 19\right)$$

at time t. Find the tangential component a_T and normal component a_N of the acceleration of the particle.

Multiple choice section

Problem 13. (5 points) True or False? In three-dimensional space $\mathbb{R}^3$...

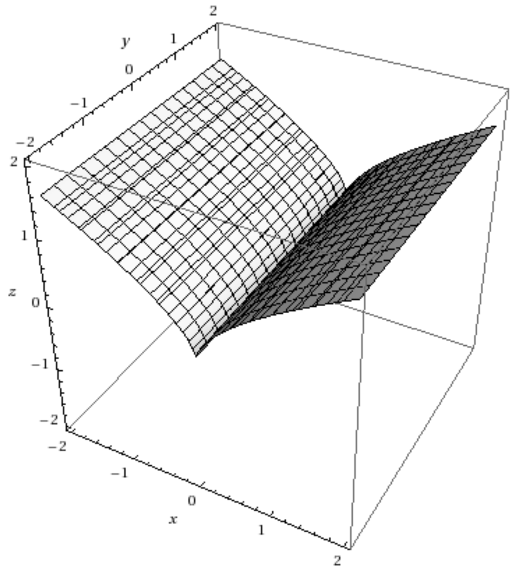
(T / F) Two planes perpendicular to some given plane must be parallel to each other.
(T / F) Two planes perpendicular to some given line must be parallel to each other.
(T / F) Two lines perpendicular to some given plane must be parallel to each other.
(T / F) Two lines perpendicular to some given line must be parallel to each other.
(T / F) Two lines parallel to some given plane must be parallel to each other.
(T / F) Two lines parallel to some given line must be parallel to each other.

Problem 14. (5 points) For any vectors $\vec{a}$ and $\vec{b}$ in $\mathbb{R}^3$, which is the following vectors is necessarily orthogonal to $\vec{a}$?

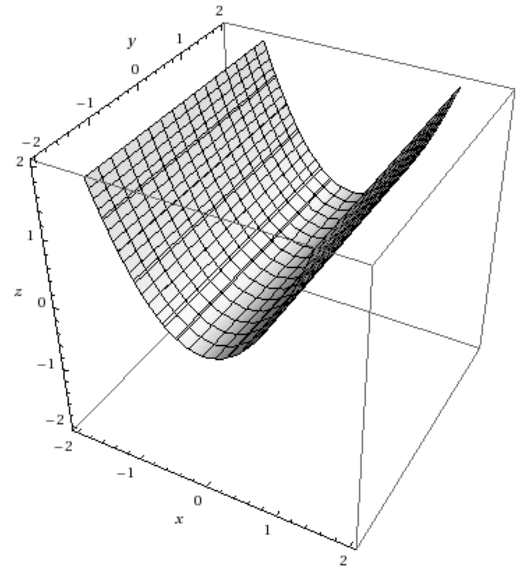
(A) $\mathrm{proj}_{\vec{b}}(\vec{a})$.
(B) $\mathrm{proj}_{\vec{a}}(\vec{b})$.
(C) $\vec{a} - \mathrm{proj}_{\vec{b}}(\vec{a})$.
(D) $\vec{b} - \mathrm{proj}_{\vec{a}}(\vec{b})$.

Problem 15. (5 points) Which of the following pictures represents the surface $x^2 = z^3$ in $\mathbb{R}^3$?

(A)



(B)



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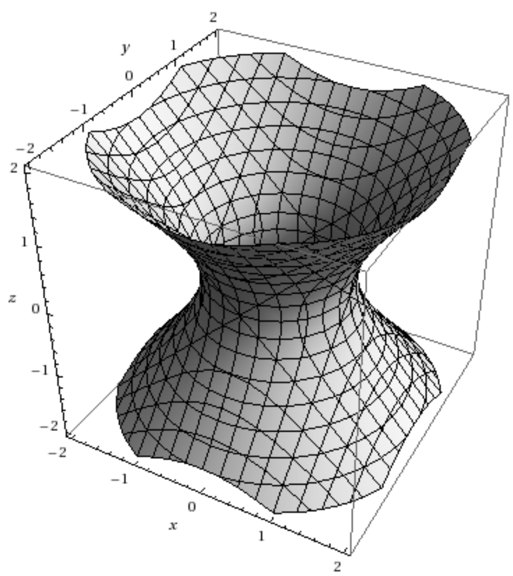

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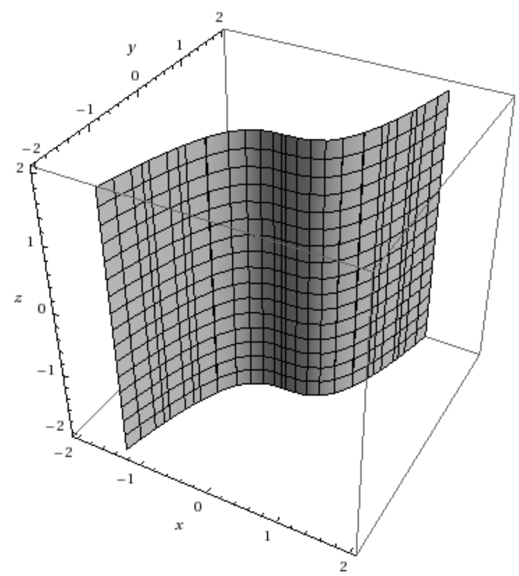


Problem 16. (5 points) The equation

$$5x^2 - 2y^2 + z^2 + 2z + 8 = 0$$

in $\mathbb{R}^3$ defines a...

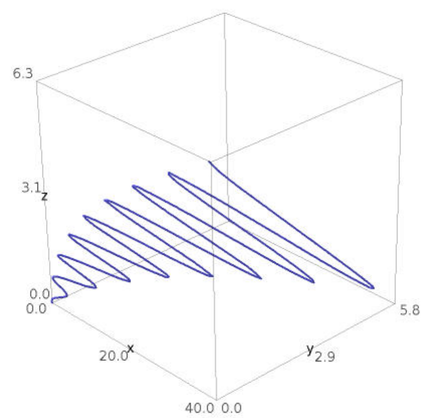
(A) Hyperboloid of one sheet.
(B) Hyperboloid of two sheets.
(C) Hyperbolic paraboloid.
(D) Hyperbolic cylinder.

Problem 17. (5 points) Choose the picture which represents the curve parametrized by

$$\vec{r}\,(t) = \left(t^2, t\sin^2 4t, t\cos^2 4t\right)$$

for $t \geq 0$.

(A)



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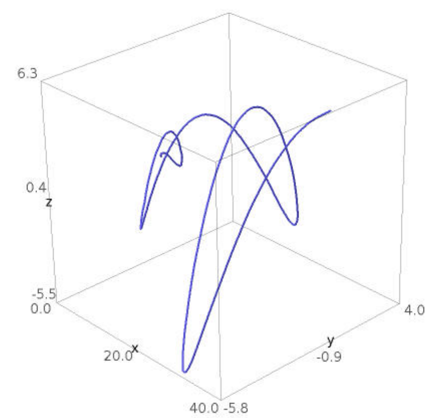

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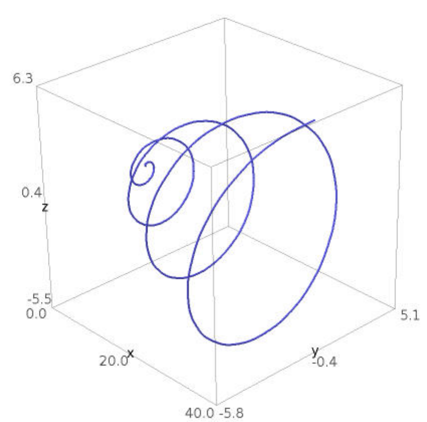

(D)