Calculus 2302A - Intermediate Calculus I

Fall 2013
Practice Midterm

Name (please print): ____

Student ID: ________________________________

- On the real exam, no calculators, electronic devices, books, or notes may be used.
- Show your work. No credit for answers without justification.
- Good luck!

(1) __________/10

(2) __________/10

(3) __________/10

(4) __________/10

(5) __________/10

(6) __________/10

(7) __________/10

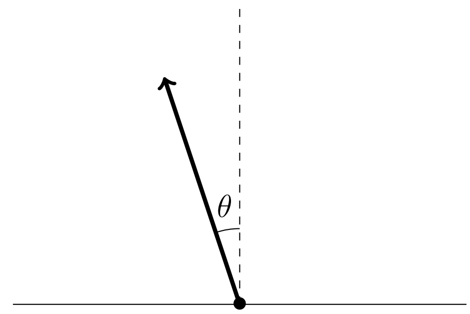
(8) __________/10

(9) __________/20

Total: __________/100

Problem 1. (10 points) A boat wants to head due north. The water current is going northeast, with a speed of 5 km/h. The boat moves at full speed, which is 50 km/h (relative to the water). In what direction should the boat head (relative to the water) so that its actual motion is due north?

Give your answer as an angle θ of deviation from the north direction, going counterclockwise. For example, $\theta = 0$ would be north and $\theta = \frac{\pi}{4}$ would be northwest.



Problem 2. (10 points) Find the distance from the point $P = (5, 2, 1)$ to the plane defined by the equation $2x + y - 4z = 3$.

Problem 3. (10 points) Find an equation for the plane that contains both the point $P = (3, 1, 5)$ and the line with parametric equations:

$$\begin{cases} x(t) = 4 - t \\ y(t) = 2t \\ z(t) = -2 + t \qquad \text{for } t \in \mathbb{R}. \end{cases}$$

Problem 4. (10 points) Find a vector equation for the line of intersection of the two planes P_1 and P_2 defined by the equations:

$$P_1 : 3x + y - 2z = 4$$
$$P_2 : x + 2y + z = 1.$$

Problem 5. (10 points) Show that the equation

$$4x^2 + 36y^2 - 72y + 9z^2 + 36z + 9 = 0$$

in $\mathbb{R}^3$ defines an ellipsoid, and find its center.

Problem 6. (10 points) Let C be a curve parametrized by a vector function $\overrightarrow{r} : \mathbb{R} \to \mathbb{R}^3$. Assume that for all $t \in \mathbb{R}$, the position vector $\overrightarrow{r}(t)$ is orthogonal to the tangent vector $\overrightarrow{r}'(t)$. Show that the curve C lies entirely on a sphere centered at the origin. (Assume that $\overrightarrow{r}$ is differentiable for all $t \in \mathbb{R}$.)

Problem 7. (10 points) Let C be the curve of intersection of the parabolic cylinder $x^2 = 2y$ and the surface $3z = xy$. Find the length of C from the origin to the point $(6, 18, 36)$.

Problem 8. (10 points) Find the curvature of the curve parametrized by $\overrightarrow{r}(t) = (t^2, 7, t)$ at the point $(1, 7, 1)$.

Multiple choice section

Problem 9. (5 points) True or False? In three-dimensional space $\mathbb{R}^3$...

(T / F) Two planes perpendicular to some given plane must be parallel to each other.
(T / F) Two planes perpendicular to some given line must be parallel to each other.
(T / F) Two lines perpendicular to some given plane must be parallel to each other.
(T / F) Two lines perpendicular to some given line must be parallel to each other.
(T / F) Two lines parallel to some given plane must be parallel to each other.
(T / F) Two lines parallel to some given line must be parallel to each other.

Problem 10. (5 points) For any vectors $\vec{a}$ and $\vec{b}$ in $\mathbb{R}^3$, which is the following vectors is necessarily orthogonal to $\vec{a}$?

(A) $\text{proj}_{\vec{b}}(\vec{a})$.
(B) $\text{proj}_{\vec{a}}(\vec{b})$.
(C) $\vec{a} - \text{proj}_{\vec{b}}(\vec{a})$.
(D) $\vec{b} - \text{proj}_{\vec{a}}(\vec{b})$.

Problem 11. (5 points) The equation

$$5x^2 - 2y^2 + z^2 + 2z + 8 = 0$$

in $\mathbb{R}^3$ defines a...

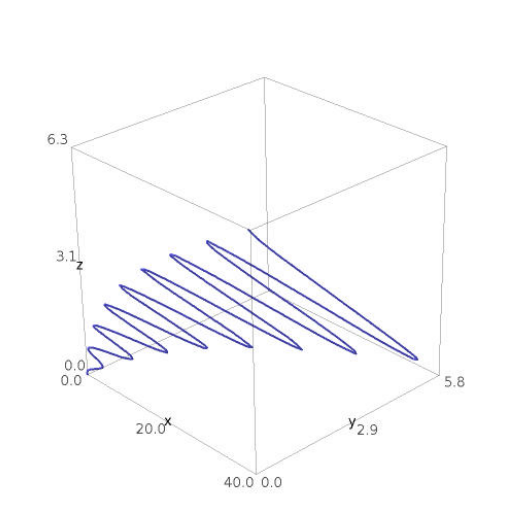
(A) Hyperboloid of one sheet.
(B) Hyperboloid of two sheets.
(C) Hyperbolic paraboloid.
(D) Hyperbolic cylinder.

Problem 12. (5 points) Choose the picture which represents the curve parametrized by

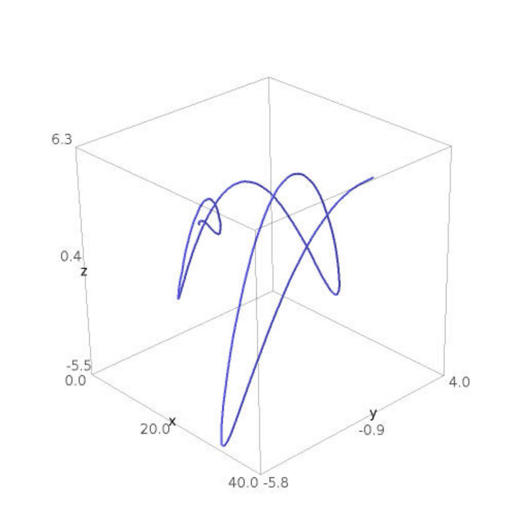
$$\overrightarrow{r}(t) = \left(t^2, t\sin^2 4t, t\cos^2 4t\right)$$

for $t \geq 0$.

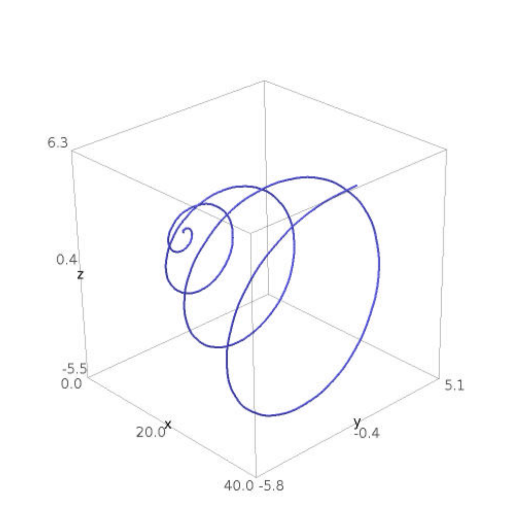
(A)



(B)



(C)



(D)