Calorimetry - 3

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Principles of Calorimetry
(Focus on Particle Physics)

Lecture 1:
i. Introduction
ii. Interactions of particles with matter (electromagnetic)
iii. Definition of radiation length and critical energy

Lecture 2:
i. Development of electromagnetic showers
ii. Electromagnetic calorimeters: Homogeneous, sampling.
iii. Energy resolution

Lecture 3:
i. Interactions of particle with matter (nuclear)
ii. Development of hadronic showers
iii. Hadronic calorimeters: compensation, resolution
Electromagnetic Shower Development

**Last lecture**

Lessons from Rossi-Heitler shower model:

\[
\begin{align*}
    t_{\text{max}} &= \frac{\ln(E_0/E_c)}{\ln 2} \\
    N_{\text{max}} &= e^{t_{\text{max}}\ln 2} = \frac{E_0}{E_c}
\end{align*}
\]

- Shower maximum at \( t_{\text{max}} \)
- Logarithm growth of \( t_{\text{max}} \) with \( E_0 \):
- \( N_{\text{max}} \propto \) energy of the primary particle

\[
N_{e^+e^-} = \frac{2}{3} \times 2 \frac{E_0}{E_c} = \frac{4}{3} \frac{E_0}{E_c}
\]

- Measured energy proportional to \( E_0 \)

\[
\alpha = \frac{N_{e^+e^-}}{E_0} = \frac{4}{3} \frac{1}{E_c} = \text{constant}
\]

Number of ions per unit of incident energy is a constant \( \Rightarrow \) absolute calibration of the calorimeter

\[
\sigma(E) \propto \frac{1}{\sqrt{E}}
\]

- Resolution improves with \( E \) (homogenous calorimeter)

- Longitudinal development scales with \( X_0 \)
- Lateral development scales with \( \rho_M \)

95% of the shower is contained laterally in a cylinder with radius \( 2\rho_M \)
Electromagnetic Shower Development

**Last lecture**

**Resolution**

\[
\frac{\sigma(E)}{E} = \frac{a}{\sqrt{E}} \oplus b \oplus \frac{c}{E}
\]

- **Statistic fluctuations**
- **Constant term (calibration, non-linearity, etc)**
- **Noise, etc**

**Sampling Calorimeter:**

- **Detectors**
- **Absorbers**

\[
N_{\text{sample}} \propto \frac{N_{e^+e^-}}{d \left[ X_0 \right]}
\]

- \( \sigma_{\text{sample}} \propto \frac{1}{\sqrt{N_{\text{sample}}} \propto \sqrt{d}} \)

The more we sample, the better is the resolution

Worst resolution than homogenous calorimeter \( \frac{\sigma(E)}{E} \propto \frac{1}{\sqrt{E}} \)
Some considerations on energy resolution

In sampling calorimeters, the distance \( d \) can increase due to multiple scattering.

\[
d_{\text{eff}} = \frac{d}{\langle \cos(\theta) \rangle} \Rightarrow \frac{\sigma(E)}{E} \propto \sqrt{\frac{d}{\langle \cos(\theta) \rangle}} \frac{1}{\sqrt{E}}
\]

For lead, \( \langle \cos(\theta) \rangle \approx 0.57 \)

Some other factors that may contribute to the energy resolution:

- Electronic noise
- ADC pedestal width
- Photodetector statistics or gain variations
- Landau tail in sampling calorimeters with gas as active element
- Pileup (more than one event within the time)
- Energy leakage
Electromagnetic Shower Development

Some considerations on energy resolution

Energy leakage

Longitudinal leakage

- EGS4 simulations

More $X_0$ needed to contain $\gamma$ initiate shower

Lateral leakage

- ~ No energy dependence
We know how to measure particles that leave most of their energies in matter via EM interaction.

But… and now? How do we measure hadrons???
Interaction of Particle with Matter

Nuclear interaction

→ Much more complex than EM interactions
→ A hadron strikes a nucleus
  → Interaction between partons
  → Excitation and breakup of the nucleus
    → Nucleus fragments
    → Production of secondary particles:
      Charged hadrons: $\pi^\pm$, p, …
      Neutral hadrons: n, $\pi^0$, …
      Charged leptons: $\mu^\pm$, …
      Neutral leptons: $\eta$
      Low energy $\gamma$, etc…

→ Total cross-section for interaction of a hadron with matter:

$$\sigma_{tot} = \sigma_{abs} + \sigma_{el} + \sigma_q$$

$\sigma_{tot}$ = total cross-section
$\sigma_{abs}$ = absorption cross-section (inelastic interaction)
$\sigma_{el}$ = elastic cross-section (hadron is preserved)
$\sigma_q$ = quasi-elastic cross-section (hadron is preserved)
Interaction of Particle with Matter

**Nuclear interaction**

→ Several processes contribute to the hadron-matter interaction

→ Only (about) half of the primary hadron energy is passed on to fast secondary particles

→ The other half is consumed in production of slow pions and other process:

→ Nuclear excitation

→ Nucleon spallation → slow neutrons

→ etc..

→ Great part of this energy is “lost” : binding energy of the nucleus production of neutrinos, etc

→ Part can be recovered: slow neutrons can interact with H atoms in active material like scintillator

For example, in lead (Pb):
- **Nuclear break-up (invisible) energy: 42%**
- **Ionization energy: 43%**
- **Slow neutrons (E_K ~ 1 MeV): 12%**
- **Low energy γ’s (E_γ ~ 1 MeV): 3%**
Hadronic shower

→ Process similar to EM shower:
  → Secondary particles interact and produces:
  → tertiary particles
  → tertiary particles interact and produces
  → …… (and so forth)
→ However, processes involved are much more complex
  → Many more particles produced
  → Multiplicity $\propto \ln(E)$  (E = energy of the primary hadron)
→ Shower ceases when hadron energies are small enough for energy loss by ionization or to be absorbed in a nuclear process.

→ The longitudinal development of the shower scales with the nuclear interaction length, $\lambda_I$:

$$\lambda_I = \frac{A}{N_A \sigma_{abs}}$$

→ The secondary particles are produced with large transverse momentum $\langle p_T \rangle > 0.35$ GeV/c Consequently, hadronic showers spread more laterally than EM showers.
Development of Hadronic Showers

**Hadronic shower**

- At energies > 1 GeV, cross-section depends little on energy:
  - \( \sigma_{abs} \approx \sigma_0 A^{0.7} \), \( \sigma_0 \approx 35 \text{ mb} \) \( \Rightarrow \)
  - \( \lambda_I \propto A^{1/3} \)

- For \( Z > 6 \) \( \Rightarrow \lambda_I > X_0 \)

### Table: Comparison of Materials

<table>
<thead>
<tr>
<th>Material</th>
<th>Z</th>
<th>A</th>
<th>( \rho ) [g/cm³]</th>
<th>( X_0 ) [g/cm²]</th>
<th>( \lambda_I ) [g/cm²]</th>
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</thead>
<tbody>
<tr>
<td>Hydrogen (gas)</td>
<td>1</td>
<td>1.01</td>
<td>0.0899 (g/l)</td>
<td>63</td>
<td>50.8</td>
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<td>Helium (gas)</td>
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<td>4.00</td>
<td>0.1786 (g/l)</td>
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<td>Beryllium</td>
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<td>9.01</td>
<td>1.848</td>
<td>65.19</td>
<td>75.2</td>
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<td>Carbon</td>
<td>6</td>
<td>12.01</td>
<td>2.265</td>
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<td>Nitrogen (gas)</td>
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<td>1.428 (g/l)</td>
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<td>2.33</td>
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<td>131.9</td>
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<td>6.8</td>
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<td>Lead</td>
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<td>6.4</td>
<td>194.0</td>
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<td>Uranium</td>
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<td>238.03</td>
<td>18.95</td>
<td>6.0</td>
<td>199.0</td>
</tr>
</tbody>
</table>

Comparing \( X_0 \) and \( \lambda_I \), we understand why Hadronic calorimeters are in general larger than EM calorimeters.
Shower profile

→ Longitudinal distribution scales with $\lambda_I$.

→ Transverse distribution depends on the longitudinal depth.
  → Initially the shower is narrow, and spreads laterally with the shower depth.

→ As in electromagnetic showers, defines a shower maximum at a position $x$ (in units of $\lambda_I$) which also depends logarithmically on energy $E$ of the primary hadron:

$$\frac{x}{\lambda_I} \equiv t_{\text{max}}(\lambda_I) \approx 0.2 \ln \left( \frac{E}{1 \text{GeV}} \right) + 0.7$$

$\Rightarrow L_{95\%}(\lambda_I) \approx t_{\text{max}} + 2.5\lambda_{\text{att}}$ is the longitudinal dimension need to contain 95% of the hadronic shower.

$$\lambda_{\text{att}} \approx \lambda_I \left( \frac{E}{1 \text{GeV}} \right)^{0.13}$$

$\lambda_{\text{att}} = \text{describes the exponential decay of the shower after } t_{\text{max}}$

→ 95% of the shower is contained within a $R < \lambda_I$ cone around the axis of the shower.
Development of Hadronic Showers

**Shower profile**

- Hadronic showers much longer than EM shower
- Also broader

![Graph showing shower profiles for different materials and energies, highlighting longitudinal energy deposit and containment.]

**Note:** $\lambda_{l}(\text{Al}) = 39.4 \text{ cm} > X_{0}(\text{Al}) = 68.9 \text{ cm}$

Usually, hadronic calorimeters are longer than EM calorimeters.
Development of Hadronic Showers

**Energy deposition**

Hadronic shower has a long longitudinal development. For 200 GeV, need $> 10 \lambda_1$ to contain 99% of the energy.

The maximum at low depth values is due to the EM component in the shower that develops more readily due to the $X_0$ dependence on $Z$ compared to $\lambda_1$:

$$X_0 \propto \frac{A}{Z^2} \ll \lambda_1 \propto A^{1/3}$$
Development of Hadronic Showers

**Energy measurement**

Based on the same principle as for the electromagnetic shower

- Shower develops until a $E_{\text{min}}$
- Energy deposition by ionization ($\pi^0 \rightarrow \gamma \gamma$ and charged hadrons) and low-energy hadronic activity (fission, neutron elastic scattering off proton, etc)

There are two components in the mechanism of energy deposition

- **Electromagnetic component**, due to $\pi^0 \rightarrow \gamma \gamma$ with subsequent EM photon interactions
- **Hadronic**

The end product is sampled and converted into signal.

The ratio between the efficiency in energy deposition due to EM interaction is and hadronic interaction is given by $e/h$
Hadronic Calorimeter (HCAL)

→ Hadronic calorimeters are usually sampling calorimeters

→ The active medium made of similar material as in EM calorimeters:
  → Scintillator (light), gas (ionization chambers, wired chambers), silicon (solid state detectors), etc

→ The passive medium is made of materials with longer interaction length $\lambda_I$
  → Iron, uranium, etc

→ Resolution is worse than in EM calorimeters (discussion in the next slides), usually in the range:

$$\frac{\sigma(E)}{E} \propto \frac{35\%-80\%}{\sqrt{E}}$$

Can be even worse depending on the goals of an experiment and compromise with other detector parameters
Hadronic Calorimeter (HCAL)

CMS hadron calorimeter

- 16 scintillator 4 mm thick plates (active material) Interleaved with 50 mm thick plates of brass

Energy resolution:

\[
\sigma(E) \propto \frac{(120\%) \oplus 5\%}{\sqrt{E}}
\]

Hadronic energy resolution compromised in favor of a much higher EM energy resolution

http://www.flickr.com/photos/naezmi/365114338/
**Hadronic Calorimeter**

### Fluctuations

#### Sampling fractions

- One can write the response of the calorimeter as:
  \[
  \pi^\pm = f_{em}e + f_hh
  \]
  - \(\pi^\pm\) = response of the calorimeter to charged pions
  - \(e\) = EM response
  - \(h\) = hadronic response
  - \(f_{em}\) = fraction of EM energy
  - \(f_h\) = fraction of hadronic energy

- The EM fraction of the shower is large (about 1/3 of the produced pions are \(\pi^0\))
- Large fluctuations in EM shower
- \(f_{em}\) depend on the energy of the primary particle

- If \(\frac{e}{h} \neq 1\) than:
  - \(\frac{\sigma(E)}{E}\) not proportional to \(\frac{1}{\sqrt{E}}\)
  - Hadron response non-linear
  - Energy deposition distribution “non Poisson”

(Ps.: hadronic means everything in the shower but the EM component)
Hadronic Calorimeter

**Fluctuations**

**Sampling fractions**

Dependence of $f_{em}$ with the energy of a primary pion
Fluctuations

Sampling fractions

→ Ideally, one wants
\[ \frac{e}{h} = 1 \]

→ But in general:
\[ \frac{e}{h} > 1 \]

because not all available hadronic energy is sampled:

→ Lost nuclear binding energy
→ neutrino energy
→ Slow neutrons, …

→ We should find a way of increasing \( h \) and at the same time decrease the EM fluctuations \( \rightarrow \) decrease \( e \)

Remember, in lead (Pb):

- **Nuclear break-up (invisible) energy**: 42%
- Ionization energy: 43%
- Slow neutrons \( (E_k \sim 1 \text{ MeV}) \): 12%
- Low energy \( \lambda \)’s \( (E_\gamma \sim 1 \text{ MeV}) \): 3%
Fluctuations

Compensation

Since the hadronic and EM energy depositions are different: \( \frac{de}{dx} \neq \frac{dh}{dx} \)

One can use the concept of the sampling calorimeter and chose appropriate passive and active media to achieve full compensation between the EM and hadronic part of the shower → increase \( h \), and slightly decrease \( e \)

→ Recover part of the invisible energy → less fluctuations in the hadronic component
→ Decrease the electromagnetic contribution → less fluctuation from the EM part of the shower

→ Select:
  → Passive medium: U, W, Pb, etc
  → Active medium: Scintillator, gas, etc
  → Thickness of the layers,
  → etc,

→ One can basically tune our calorimeter to “compensate”
Fluctuations

Compensation

- Full compensation can be achieved with
  - **High Z material as absorber**
    - Remember, e.g., photoelectric effect goes with $Z^5$, therefore large part of the EM shower will be deposit in the absorber decreasing the EM sampling fraction (less energy deposition in the active medium)
  - Tuning the thickness of the absorber and active layer
    - For the same length to have shower containment in the calorimeter, tune the thickness of the absorber and active media such the EM sampling fraction decreases due to the same reason discussed above
  - High interact absorber that can partially recover the invisible hadronic energy via nuclear and collisions processes.
Hadronic Calorimeter

**Fluctuations**

**Compensation**

→ e.g., $^{238}$U as **passive** and **scintillator** as **active** media.

$^{238}$U:

→ Absorber with high Z → decreases e

→ Slow neutrons induces fission in the $^{238}$U

→ Fission energy compensates loss due to “invisible” energy carried by the slow neutrons

→ Slow neutron can be captured nucleus of $^{238}$U which emits a low energy $\gamma$’s

→ Can further recover the “invisible” energy

**Scintillator:**

→ Slow neutrons also loose their kinetic energy via elastic collisions with nucleus

→ The lighter the nuclei, more energy transferred to the active medium

→ Scintillators are reach in Hydrogen
Example of Compensate Calorimeter

Compensation

ZEUS Uranium-Scintillator detector

→ 78 modules made up of Scintillator-Uranium plates
→ Absorber layer ($^{238}\text{U}$) : 3.3 mm thick
→ Scintillator layer: 2.6 mm thick
→ $1X_0 (0.04\lambda_1)$ throughout the entire calorimeter
Hadronic Calorimeter

Example of Compensate Calorimeter

Compensation

\[ e/h \] ration for incident pions at different energies \( E_k \)
Hadronic Calorimeter

Example of Compensate Calorimeter

ZEUS

Hadronic energy resolution:

\[
\frac{\sigma(E)}{E} = \frac{35\%}{\sqrt{E}}
\]

However, relatively low EM energy resolution

\[
\frac{\sigma(E)}{E} = \frac{18\%}{\sqrt{E}}
\]

Reason: \( I X_\theta \) required for compensation and practical limitations in tuning scintillator thickness (2 to 3 mm) (could be improved using 1mm diameter scintillator fibers)
Particle Flow Concept

Compensation is not the only method to improve the hadronic energy resolution. The key element is to reduce fluctuations. This can be done using the following recipe:

For charged particles with energy up to $\sim 100$ GeV, tracking detectors measure momentum more accurately than calorimeters. The following considerations are then used for the reconstruction of the 4-momentum of a particle:

- Tracks can be associated to the initial point of a shower in a calorimeter
  - EM showers with track association are considered as initiated by electrons or positrons
  - Energy deposition due to minimum ionizing particles in the calorimeter with track association are considered as muons
  - Hadronic showers with track association are considered charged pions
  - Four-momentum of the particle is then reconstructed using full tracking information
  - EM showers with no track association are considered as initiated by photons
  - Hadronic showers with no track association are considered as initiated by neutral hadrons
Hadronic Calorimeter

Fluctuations - Other methods to improve resolution

Particle Flow Concept

Particle flow scheme:

> Tracker (tracking detector to reconstruct charge particles) \((<65\%)\) of a jet

> ECAL for \(\gamma\) reconstruction \((<25\%)\)

> ECAL+HCAL for \(h^0 (\pi^0, \text{etc})\) reconstruction \((<10\%)\)
**Fluctuations - Other methods to improve resolution**

**Particle Flow Concept**

**Considerations:**

- All particles in a event have to be measured
- Calorimeters (EM and hadronic) have to be highly segmented for tracking association
- Large acceptance (angular coverage) necessary for event containment
- Compensation not necessary, though desirable if feasible.

**Advantage over pure compensation:** Can deliver high electromagnetic energy resolution, and at same time considerable improve the hadronic energy resolution.
Hadronic Calorimeter

Fluctuations - Other methods to improve resolution

Particle Flow Concept

Example:

→ Development of a dedicated detector using the particle flow concept:
The International electron-positron linear collider (ILC)

High granularity;
Steal (absorber)/scintillator tile (active) plates.

Note: Prototyping phase; other materials, geometries and technologies under consideration
**Hadronic Calorimeter**

**Fluctuations - Other methods to improve resolution**

**Particle Flow Concept**
**The International Linear Collider**

Designed hadronic energy resolution:

$$\frac{\sigma(E)}{E} \approx 30\% / \sqrt{E}$$

Typical event to be observed at ILC in searching for the Higgs boson:

- **Missing mass peak** or $H \rightarrow b\bar{b}$
- Want to separate from $WW$, $ZZ$

Impact of higher energy resolution on the reconstruction of two jets (particle showers) $\rightarrow$ jet separation
Resolution of some electromagnetic calorimeters (PDG, pdg.lbl.gov)

<table>
<thead>
<tr>
<th>Technology (Experiment)</th>
<th>Depth</th>
<th>Energy resolution</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>NaI(Tl) (Crystal Ball)</td>
<td>20$X_0$</td>
<td>2.7%/E$^{1/4}$</td>
<td>1983</td>
</tr>
<tr>
<td>Bi$_4$Ge$<em>3$O$</em>{12}$ (BGO) (L3)</td>
<td>22$X_0$</td>
<td>2%/\sqrt{E} + 0.7%</td>
<td>1993</td>
</tr>
<tr>
<td>CsI (KTeV)</td>
<td>27$X_0$</td>
<td>2%/\sqrt{E} + 0.45%</td>
<td>1996</td>
</tr>
<tr>
<td>CsI(Tl) (BaBar)</td>
<td>16–18$X_0$</td>
<td>2.3%/E$^{1/4}$ + 1.4%</td>
<td>1999</td>
</tr>
<tr>
<td>CsI(Tl) (BELLE)</td>
<td>16$X_0$</td>
<td>1.7% for $E_\gamma &gt; 3.5$ GeV</td>
<td>1998</td>
</tr>
<tr>
<td>PbWO$_4$ (PWO) (CMS)</td>
<td>25$X_0$</td>
<td>3%/\sqrt{E} + 0.5% + 0.2/E</td>
<td>1997</td>
</tr>
<tr>
<td>Lead glass (OPAL)</td>
<td>20.5$X_0$</td>
<td>5%/\sqrt{E}</td>
<td>1990</td>
</tr>
<tr>
<td>Liquid Kr (NA48)</td>
<td>27$X_0$</td>
<td>3.2%/\sqrt{E} + 0.42% + 0.09/E</td>
<td>1998</td>
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<td>Scintillator/depleted U (ZEUS)</td>
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<td>18%/\sqrt{E}</td>
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<td>Scintillator fiber/Pb spaghetti (KLOE)</td>
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<td>1996</td>
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</table>
Summary

Summary-2

Lessons we learnt in these lectures:

- Building your calorimeter to measure particles in Particle Physics:

1) Identify your goal:
   → What do you want to measure? (Physics)
   → What energy do you want to measure? (dynamic range)
   → How much do you have to spend? (cost)

2) Identify the proper material
   → Want to full contain the particle in the calorimeter
   → Want to minimize fluctuations for better energy measurement
   → Want low noise environment (remember the extra terms in the energy resolution)
   → Want statistics for accuracy in your results

3) Have you decided? Then gather a group of people and build your prototype.