Principles of Calorimetry
(Focus on Particle Physics)

Lecture 1:
  i.  Introduction
  ii. Interactions of particles with matter (electromagnetic)
  iii. Definition of radiation length and critical energy

Lecture 2:
  i. Development of electromagnetic showers
  ii. Electromagnetic calorimeters: Homogeneous, sampling.
  iii. Energy resolution

Lecture 3:
  i. Interactions of particle with matter (nuclear)
  ii. Development of hadronic showers
  iii. Hadronic calorimeters: compensation, resolution
Interactions of Photons

Last lecture:

\[ \sigma = \sigma_{\text{p.e.}} + \sigma_{\text{Comp}} + \sigma_{\text{pair}} + \ldots \]

p.e. = Photoelectric effect
Comp = Compton scattering
pair = \( e^+ e^- \) pair production

Energy range versus \( Z \) for more likely process:

http://pdg.lbl.gov
Interactions of Particles with Matter

Interactions of Electrons

Ionization (Fabio Sauli’s lecture)

For "heavy" charged particles ($M \gg m_e$: p, K, π, μ), the rate of energy loss (or stopping power) in an inelastic collision with an atomic electron is given by the Bethe-Block equation:

$$\frac{dE}{dx} = 2\pi N \rho^2 m_0 c^2 \left( \frac{Z v^2}{\beta^2} \right) \ln \left( \frac{2m_0 c^2 \beta^2 \gamma W_{\text{max}}}{I^2} \right) - 2\beta^2 - \delta(\beta \gamma) - 2 \left( \frac{C}{Z} \right)$$

$\delta(\beta \gamma)$: density-effect correction

$C$: shell correction

$z$: charge of the incident particle

$\beta = \frac{v}{c}$ of the incident particle; $\gamma = (1 - \beta^2)^{-1/2}$

$W_{\text{max}}$: maximum energy transfer in one collision

$I$: mean ionization potential

http://pdg.lbl.gov
Interactions of Electrons

Ionization

For electrons and positrons, the rate of energy loss is similar to that for “heavy” charged particles, but the calculations are more complicate:

- Small electron/positron mass
- Identical particles in the initial and final state
- Spin $\frac{1}{2}$ particles in the initial and final states

\[
-\frac{dE}{dx} = 2\pi N_A r_e^2 m_e c^2 \frac{Z}{A} \beta^2 \left[ \ln \left( \frac{k^2 (k+2)}{2 I^2 / m_e c^2} \right) - F(k, \beta, \gamma) - \delta - 2 \frac{C}{Z} \right] \left[ \text{MeV cm} \div g \right]
\]

$k = E_k / m_e c^2$: reduced electron (positron) kinetic energy

$F(k, \beta, \gamma)$ is a complicate equation

However, at high incident energies ($\beta \approx 1$) $\Rightarrow F(k) \approx \text{constant}$
Interactions of Electrons

Ionization

At this high energy limits \((\beta \approx 1)\), the energy loss for both “heavy” charged particles and electrons/positrons can be approximate by

\[
-\frac{dE}{dx} \propto \left[ 2 \ln \left( \frac{2m_e c^2}{I} \right) + A \ln \gamma - B \right]
\]

Where,

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>electrons</td>
<td>3</td>
<td>1.95</td>
</tr>
<tr>
<td>heavy charged particles</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

The second terms indicates that the rate of relativistic rise for electrons is slightly smaller than for heavier particles. **This provides a criterion for identification between charge particles of different masses.**
Interactions of Electrons

Bremsstrahlung (breaking radiation)

A particle of mass $m_i$ radiates a real photon while being decelerated in the Coulomb field of a nucleus with a cross section given by:

$$\frac{d\sigma}{dE} \propto \frac{Z^2}{m_i^2} \ln E$$

$m_i^2$ factor expected since classically radiation $\propto a_i^2 = \left(\frac{F}{m_i}\right)^2$

→ Makes electrons and positrons the only significant contribution to this process for energies up to few hundred GeV’s.

$$\left(\frac{d\sigma}{dE}\right)_e / \left(\frac{d\sigma}{dE}\right)_\mu = \left(\frac{m_\mu}{m_e}\right)^2 \approx 37 \times 10^3$$
Interactions of Particles with Matter

Interactions of Electrons

Bremsstrahlung

The rate of energy loss for $k >> 137/Z^{1/3}$ is given by:

$$- \frac{dE}{dx} \approx r_e^2 4\alpha Z^2 \frac{N_A}{A} \left( \ln \frac{183}{Z^{1/3}} \right) E$$

Recalling from pair production

$$X_0 = \frac{A}{N_A} \frac{1}{r_e^2 4\alpha Z^2 \ln \left( \frac{183}{Z^{1/3}} \right)} \quad \Rightarrow \quad \frac{dE}{dx} = -\frac{E}{X_0} \quad \Rightarrow \quad E(x) = E_0 e^{-\frac{x}{X_0}}$$

The radiation length $X_0$ is the layer thickness that reduces the electron energy by a factor $e$ ($\approx 63\%$)
Interactions of Electrons

Bremsstrahlung

Radiation loss in lead.

Figure 27.10: Fractional energy loss per radiation length in lead as a function of electron or positron energy. Electron (positron) scattering is considered as ionization when the energy loss per collision is below 0.255 MeV, and as Möller (Bhabha) scattering when it is above. Adapted from Fig. 3.2 from Messel and Crawford, *Electron-Photon Shower Distribution Function Tables for Lead, Copper, and Air Absorbers*, Pergamon Press, 1970. Messel and Crawford use $X_0(\text{Pb}) = 5.82$ g/cm$^2$, but we have modified the figures to reflect the value given in the Table of Atomic and Nuclear Properties of Materials ($X_0(\text{Pb}) = 6.37$ g/cm$^2$).
Interactions of Electrons

Bremsstrahlung and Pair production

→ Note that the mean free path for photons for pair production is very similar to $X_0$ for electrons to radiate Bremsstrahlung radiation:

$$\lambda_{pair} = \frac{9}{7} X_0$$

→ This fact is not coincidence, as pair production and Bremsstrahlung have very similar Feynman diagrams, differing only in the directions of the incident and outgoing particles (see Fernow for details and diagrams).

In general, an electron-positron pair will each subsequently radiate a photon by Bremsstrahlung which will produce a pair and so forth → shower development
Interactions of Particles with Matter

Interactions of Electrons

Bremsstrahlung (Critical Energy)

Another important quantity in calorimetry is the so called critical energy. One definition is that it is the energy at which the loss due to radiation equals that due to ionization. PDG quotes the Berger and Seltzer:

\[ E_c \approx \frac{dE}{dx} (E_c)_{\text{Brem}} = \frac{dE}{dx} (E_c)_{\text{ion}} \]

\[ E_c \approx \frac{800 \text{MeV}}{(Z + 1.2)} \]

Other definition \( \rightarrow \) Rossi

\[ Z_0 = 12.86 \text{ g cm}^{-2} \]

\[ E_c = 19.63 \text{ MeV} \]

Figure 27.12: Two definitions of the critical energy \( E_c \).
Interactions of Particles with Matter

Summary of the basic EM interactions

- \( \text{e}^+ / \text{e}^- \)
  - Ionisation

- Bremsstrahlung

- P.e. effect

- Comp. effect

- Pair production

\[ \frac{dE}{dx} \]

\[ E \]

\[ Z \]

\[ Z(Z+1) \]

\[ g \]

\[ s \]

\[ s \]

\[ Z^5 \]

\[ Z \]

\[ Z(Z+1) \]
Electromagnetic Shower Development

Detecting a signal:

→ The contribution of an electromagnetic interaction to energy loss usually depends on the energy of the incident particle and on the properties of the absorber

→ At “high energies” ($>\sim 10\ MeV$):
  → electrons lose energy mostly via Bremsstrahlung
  → photons via pair production

→ Photons from Bremsstrahlung can create an electron-positron pair which can radiate new photons via Bremsstrahlung in a process that last as long as the electron (positron) has energy $E > E_c$

→ At energies $E < E_c$, energy loss mostly by ionization and excitation

→ Signals in the form of light or ions are collected by some readout system

Building a detector

→ $X_0$ and $E_c$ depends on the properties of the absorber material

→ Full EM shower containment depends on the geometry of the detector
Electromagnetic Shower Development

A simple shower model **(Rossi-Heitler)**

Considerations:

- Photons from bremsstrahlung and electron-positron from pair production produced at angles $\theta = mc^2/E$ ($E$ is the energy of the incident particle) → jet character

Assumptions:

- $\lambda_{pair} \approx X_0$
- Electrons and positrons behave identically
- Neglect energy loss by ionization or excitation for $E > E_c$
- Each electron with $E > E_c$ gives up half of its energy to bremsstrahlung photon after $1X_0$
- Each photon with $E > E_c$ undergoes pair creation after $1X_0$ with each created particle receiving half of the photon energy
- Shower development stops at $E = E_c$
- Electrons with $E < E_c$ do not radiate → remaining energy lost by collisions

*B. Rossi*, High Energy Particles, New York, Prentice-Hall (1952)

A simple shower model

Shower development:

Start with an electron with $E_0 \gg E_c$

$\rightarrow$ After $1X_0$: 1 $e^-$ and 1 $\gamma$, each with $E_0/2$

$\rightarrow$ After $2X_0$: 2 $e^-$, 1 $e^+$ and 1 $\gamma$, each with $E_0/4$

$\rightarrow$ Number of particles increases exponentially with $t$

$\rightarrow$ Equal number of $e^+$, $e^-$, $\gamma$

$\rightarrow$ Depth at which the energy of a shower particle equals some value $E'$

$\rightarrow$ Number of particles in the shower with energy $> E'$

Maximum number of particles reached at $E = E_c$ →

$N(t) = 2^t = e^{t \ln 2}$

$E(t) = \frac{E_0}{2^t}$

$t(E') = \frac{\ln \left( \frac{E_0}{E'} \right)}{\ln 2}$

$N(E > E') = \frac{1}{\ln 2} \frac{E_0}{E'}$

$t_{\max} = \frac{\ln \left( \frac{E_0}{E_c} \right)}{\ln 2}$

$N_{\max} = e^{t_{\max} \ln 2} = \frac{E_0}{E_c}$
Electromagnetic Shower Development

A simple shower model

Concepts introduce with this simple mode:

→ Maximum development of the shower (multiplicity) at $t_{\text{max}}$

→ Logarithm growth of $t_{\text{max}}$ with $E_0$ :

  → implication in the calorimeter longitudinal dimensions

→ Linearity between $E_0$ and the number of particles in the shower
Electromagnetic Shower Development

A simple shower model

What about the energy measurement?

Assuming, say, energy loss by ionization

→ Counting charges:

→ Total number of particles in the shower:

\[ N_{all} = \sum_{t=0}^{t_{max}} 2^t = 2 \times 2^{t_{max}} - 1 \approx 2 \times 2^{t_{max}} = 2 \frac{E_0}{E_c} \]

→ Total number of charge particles (\(e^+\) and \(e^-\) contribute with 2/3 and \(\gamma\) with 1/3)

\[ N_{e^+e^-} = \frac{2}{3} \times 2 \frac{E_0}{E_c} = \frac{4}{3} \frac{E_0}{E_c} \]

→ Measured energy proportional to \(E_0\)
A simple shower model

What about the energy resolution?

Assuming Poisson distribution for the shower statistical process:

\[
\frac{\sigma(E)}{E} = \frac{1}{\sqrt{N_{e^+e^-}}} = \frac{\sqrt{3E_c}}{2\sqrt{E}}
\]

Resolution improves with E

Example: For lead (Pb), \( E_c \approx 6.9 \text{ MeV} \):

\[
\frac{\sigma(E)}{E} = 7.2\% = \frac{7.2\%}{\sqrt{E \text{ [GeV]}}}
\]

More general term:

\[
\frac{\sigma(E)}{E} = \frac{a}{\sqrt{E}} \oplus \frac{b}{E} \oplus \frac{c}{E}
\]

- Noise, etc
- Statistic fluctuations
- Constant term (calibration, non-linearity, etc)
A simple shower model

Simulation of the energy deposit in copper as a function of the shower depth for incident electrons at 4 different energies showing the logarithmic dependence of $t_{\text{max}}$ with $E$.

EGS4* (electron-gamma shower simulation)

*EGS4 is a Monte Carlo code for doing simulations of the transport of electrons and photons in arbitrary geometries.
A simple shower model

Though the model has introduced correct concepts, it is too simple:

→ Discontinuity at $t_{\text{max}}$ : shower stops → no energy dependence of the cross-section

→ Lateral spread → electrons undergo multiple Coulomb scattering

→ Difference between showers induced by $\gamma$ and electrons

→ $\lambda_{\text{pair}} = \frac{9}{7} X_0$

→ Fluctuations : Number of electrons (positrons) not governed by Poisson statistics.
Electromagnetic Shower Development

**Shower Profile**

→ Longitudinal development governed by the radiation length $X_0$

→ Lateral spread due to electron undergoing multiple Coulomb scattering:
  
  → About 90% of the shower up to the shower maximum is contained in a cylinder of radius $< 1X_0$
  
  → Beyond this point, electrons are increasingly affected by multiple scattering
  
  → Lateral width scales with the Molière radius $\rho_M$

\[
\rho_M = X_0 \frac{E_s}{E_c} \left[ \frac{g}{cm^2} \right], \quad E_s \approx 21\,\text{MeV}
\]

95% of the shower is contained laterally in a cylinder with radius $2\rho_M$
Electromagnetic Shower Development

Shower profile

From previous slide, one expects the longitudinal and transverse developments to scale with $X_0$:

- Longitudinal development:
  
  $\rho_M$ less dependent on $Z$ than $X_0$:

  \[ X_0 \propto A/Z^2, \quad E_c \propto 1/Z \implies \rho_M \propto A/Z \]
Electromagnetic Shower Development

**Shower profile**

Different shower development for photons and electrons

At increasingly depth, photons carry larger fraction of the shower energy than electrons
Energy deposition

The fate of a shower is to develop, reach a maximum, and then decrease in number of particles once $E_0 < E_c$

Given that several processes compete for energy deposition at low energies, it is important to understand how the fate of the particles in a shower.

→ Most of energy deposition by low energy $e^\pm$'s.
Electromagnetic Calorimeters

**Homogeneous Calorimeters**

→ Only one element as passive (shower development) and active (charge or light collection) material

→ Combine short attenuation length with large light output → **high energy resolution**

→ Used exclusively as electromagnetic calorimeter

→ Common elements are: NaI and BGO (bismuth germanate) scintillators, scintillating, glass, lead-glass blocks (Cherenkov light), liquid argon (LAr), etc

<table>
<thead>
<tr>
<th>Material Properties</th>
<th>X₀(cm)</th>
<th>ρₓ(cm)</th>
<th>λₓ(cm)</th>
<th>light</th>
<th>energy resolution (%)</th>
<th>experiments</th>
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<tbody>
<tr>
<td>NaI</td>
<td>2.59</td>
<td>4.5</td>
<td>41.4</td>
<td>scintillator</td>
<td>2.5/E¹/₄</td>
<td>crystal ball</td>
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<tr>
<td>CsI(Tl)</td>
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<td>3.8</td>
<td>36.5</td>
<td>scintillator</td>
<td>2.2/E¹/₄</td>
<td>CLEO, BABAR, BELLE</td>
</tr>
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<td>Lead glass</td>
<td>2.6</td>
<td>3.7</td>
<td>38.0</td>
<td>cerenkov</td>
<td>5/E¹/₂</td>
<td>OPAL, VENUS</td>
</tr>
</tbody>
</table>

λₓ = nuclear absorption length
Electromagnetic Calorimeters

Homogeneous Calorimeters

BaBar EM Calorimeter

→ CsI as active/passive material
→ Total of ~6500 crystals

\[ \frac{\sigma_E}{E} = \frac{(2.32 \pm 0.30)\%}{\sqrt{E} (\text{GeV})} \oplus (1.85 \pm 0.12)\% \]
Electromagnetic Calorimeters

**Sampling Calorimeters**

- Consists of two different elements, normally in a sandwich geometry:
  - Layers of Active material (collection of signal) – gas, scintillator, etc
  - Layers of passive material (shower development)

- Segmentation allows measurement of spatial coordinates

- Can be very compact → simple geometry, relatively cheap to construct

- Sampling concept can be used in either electromagnetic or hadronic calorimeter

- Only part of the energy is sampled in the active medium.
  - Extra contribution to fluctuations
Electromagnetic Calorimeters

**Sampling Calorimeters**

→ Typical elements:

- Passive: Lead, W, U, Fe, etc
- Active: Scintillator slabs, scintillator fibers, silicon detectors, LAr, LXe, etc.

Typical Energy Resolution

\[
\frac{\sigma_E}{E} = \frac{(7.5 - 25)\%}{\sqrt{E}} \quad \text{EM}
\]

\[
\frac{\sigma_E}{E} = \frac{(35 - 80)\%}{\sqrt{E}} \quad \text{hadronic}
\]
**Sampling Calorimeters**

**ATLAS LAr Accordion Calorimeter**

Ionization chamber: 1 GeV E-deposit $\rightarrow$ $5 \times 10^6$ e$^-$

Accordion shape $\Rightarrow$ Complete $\Phi$ symmetry without azimuthal cracks $\Rightarrow$ better acceptance.

Test beam results, $e^-$ 300 GeV (ATLAS TDR)

$$\frac{\sigma(E)}{E} \approx 10\% \oplus 0.7\%$$

Spatial and angular uniformity $\approx 0.5\%$

Spatial resolution $\approx 5\text{mm} / E^{1/2}$
Electromagnetic Calorimeters

Sampling Calorimeters

Sampling fraction:

The number of particles we see: \( N_{\text{sample}} \)

\[
N_{\text{sample}} \propto \frac{N_{e^+e^-}}{d \left[ X_0 \right]}
\]

d = distance between active plates

Sampling fluctuation:

\[
\frac{\sigma_{\text{sample}}}{E} \propto \frac{1}{\sqrt{N_{\text{sample}}}} \propto \frac{\sqrt{d}}{\sqrt{E}}
\]

The more we sample, the better is the resolution