Chapter 3

Kinematics in Two Dimensions; Vectors
• Vectors and Scalars
• Addition of Vectors – Graphical Methods (One and Two-Dimension)
• Multiplication of a Vector by a Scalar
• Subtraction of Vectors – Graphical Methods
• Adding Vectors by Components
• Projectile Motion
• Projectile Motion Is Parabolic
• Relative Velocity (Relative Motion)
Assignment 2

Textbook (Giancoli, 6th edition). Due on Oct. 6th.


3. A projectile, fired with unknown initial velocity, lands 20.0 s later on the side of a hill, 3000 m away horizontally and 450 m vertically above its starting point. (Ignore any effects due to air resistance.)

   (a) What is the vertical component of its initial velocity?
   (b) What is the horizontal component of its initial velocity?
   (c) What was its maximum height above its launch point?
   (d) As it hit the hill, what speed did it have and what angle did its velocity make with the vertical?

(Re-)Recalling Last Two Lectures
Adding Vectors by Components

\[
\vec{V} = \vec{V}_x + \vec{V}_y
\]

\[
(\vec{V})_x = V_x
\]

\[
(\vec{V})_y = V_y
\]

\[
V_x = V \cos \theta
\]

\[
V_y = V \sin \theta
\]

\[
|\vec{V}| = V = \sqrt{V_x^2 + V_y^2}
\]

\[
\theta = \sin^{-1} \left( \frac{V_y}{V} \right) \quad \theta = \tan^{-1} \left( \frac{V_y}{V_x} \right)
\]

\[
\theta = \cos^{-1} \left( \frac{V_x}{V} \right)
\]
Adding Vectors by Components

Note also that in general:

If $\vec{A}$ and $\vec{B}$ are two arbitrary vectors and $\vec{C}$ is the result of their addition, then:

$$\vec{C} = \vec{A} + \vec{B}$$

$C = |\vec{C}|$  \hspace{1cm}  $B = |\vec{B}|$  \hspace{1cm}  $A = |\vec{A}|$

$$C^2 = \sqrt{A^2 + B^2}$$
Adding Vectors by Components

Representing a vector by its components:

We can define a vector using the following representation:

\[ \vec{v} = (v_x, v_y) \] \hspace{1cm} (3.17)

Where in general:

\[(a, b) = (x \text{ component, } y \text{ component})\]

Example:

\[ \vec{c} = (-1.0, 5.0) \]

Tell me that

\[ c_x = -1.0 \]
\[ c_y = 5.0 \]
Adding Vectors by Components

More Comments:

If \( \vec{C} \) is a vector and \( C \) a scalar (for example, can be the magnitude of the vector), then it is wrong to write

\[
\vec{C} = C
\]

Example:

\[
\vec{C} = 43.0
\]
Projectile Motion

We can then analyze the motion in the *x* and *y* direction separately using the equations of motion we obtained in chapter 2 (only vertical acceleration \( g \) in *y* direction):

Horizontal motion:
\[
\begin{align*}
v_x &= v_{x0} + at \\
x &= x_0 + v_{x0}t + \frac{1}{2}a_xt^2 \\
v_x^2 &= v_{x0}^2 + 2a_x(x - x_0)
\end{align*}
\]

Vertical motion:
\[
\begin{align*}
v_y &= v_{y0} + gt \\
y &= y_0 + v_{y0}t + \frac{1}{2}gt^2 \\
v_y^2 &= v_{y0}^2 + 2g(y - y_0)
\end{align*}
\]

**Note:** The sign of the acceleration \( g \) in the above Eqs. depends on your choice of orientation for the axis *y*. You will have to rewrite the above equations using \(-g\) instead of \( g \) if you decide to choose \(+y\) pointing upwards as in the figure.
Projectile Motion

If an object is launched at an initial angle of $\theta_0$ with the horizontal, the analysis is similar except that the initial velocity has a vertical component.

You can use equations the equations from previous slide to develop your problem in the very same way as when $\theta_0 = 0$.

\[
\begin{align*}
V_{x0} &= V_0 \cos \theta_0 \\
V_{y0} &= V_0 \sin \theta_0
\end{align*}
\]

\[
\begin{align*}
V_x &= V \cos \theta \\
V_y &= V \sin \theta
\end{align*}
\]
Important Note

You might consider a problem where an object is subject to an acceleration other than that of gravity. The problem is again solved using the method of components where now the acceleration vector can also be represented by its components on the x and y axes:

\[ \vec{a} = \vec{a}_x + \vec{a}_y \quad ; \quad (\vec{a})_x = a_x \quad ; \quad (\vec{a})_y = a_y \]

The resultant acceleration is the addition of \( \vec{a} \) and \( \vec{g} \)

\[ \vec{A} = \vec{a} + \vec{g} \]

\[ \vec{A}_y = \vec{a}_y + \vec{g} \quad \quad \quad \vec{A}_x = \vec{a}_x \]

\[ (\vec{A})_x = (\vec{a})_x + (\vec{g})_x = a_x + 0 = a_x \]

\[ (\vec{A})_y = (\vec{a})_y + (\vec{g})_y = a_y - g \]
Important Note

The equations of motion in x and y are then written as:

**horizontal motion**

\[ v_x = v_{x0} + a_x t \]
\[ x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2 \]
\[ v_x^2 = v_{x0}^2 + 2a_x(x - x_0) \]

**vertical motion**

\[ v_y = v_{y0} + (a_y - g)t \]
\[ y = y_0 + v_{y0} t + \frac{1}{2} (a_y - g)t^2 \]
\[ y - y_0 = v_{y0} t + \frac{1}{2} (a_y - g)t^2 \]

Note that the minus sign in \((\vec{A})_y = (\vec{a})_y + (\vec{g})_y = a_y - g\) is due to choice of having the y axis pointing upwards.

If there is **NO gravity**, consider \(g = 0\) in the above equations.
Projectile Motion is Parabolic

It is not difficult to show that whenever we assume the acceleration of gravity to be constant and ignore the air resistance, then the vertical displacement $y$ can be written as a function of the horizontal displacement $x$. Using the equations of motion 2.10, 2.13, 2.14 and 2.16-2.18, we can show that:

$$y = Ax - Bx^2$$

I leave for you to obtain the values of $A$ and $B$ in terms of the initial angle $\theta_0$, initial velocity $v_0$ and acceleration of gravity $g$ (assume $y$ axis pointing upwards).
Relative Velocity

We have seen that it is important to choose a reference frame where to define quantities such as velocity, position, etc.

An example was that of a person walking along a train which is moving relative to the ground. The velocity YOU measure of this person depends on whether you are on the train or standing on the ground.

It is common to place yourself at the origin of your coordinate system (reference frame) when you are the one performing the measurement.
Relative Velocity

This is an example of relative velocity. If the train moves with velocity $\vec{v}_{TG}$ relative to the ground, and the person moves with velocity $\vec{v}_{PT}$ relative to the train, then, you will measure the velocity $\vec{v}_{PY}$ relative to you as:

**You are on the train:**

\[
\vec{v}_{PY} = \vec{v}_{PT} + \vec{v}_{TY} = \vec{v}_{PT} + 0 = \vec{v}_{PT}
\]

Where $\vec{v}_{TY}$ is the velocity of the train relative to you (zero in this case).

(ground is moving this way relative to you)
Relative Velocity

You are on the ground:

\[ \vec{v}_{PY} = \vec{v}_{PT} + \vec{v}_{TY} = \vec{v}_{PT} + \vec{v}_{TG} \]
Relative Velocity

In a more general case, you could be moving in a car with velocity $\vec{v}_{YG}$ relative to the ground. In this case you will measure:

First note that:  
\[ \vec{v}_{YG} = -\vec{v}_{GY} \]  
(3.17)

Then:  
\[ \vec{v}_{PY} = \vec{v}_{PT} + \vec{v}_{TG} + \vec{v}_{GY} \]  
, or  
\[ \vec{v}_{PY} = \vec{v}_{PT} + \vec{v}_{TG} - \vec{v}_{YG} \]  
(3.18)
Relative Velocity

The previous example addressed a one dimensional problem. However, it can be easily generalized to two (or three) dimensional problems thanks to the fact that we have been using vectors as a method (tool) to analyze motion.

Let \( \vec{v}_{BW} \) be the velocity of a boat relative to the river water and \( \vec{v}_{WS} \) the velocity of the water relative to the shore. Then the velocity \( \vec{v}_{BS} \) of the boat relative to the shore will be given by:

\[
\vec{v}_{BS} = \vec{v}_{BW} + \vec{v}_{WS} \tag{3.19}
\]

You can then use the component method for vector addition to obtain \( \vec{v}_{BS} \).
Relative Velocity

Note that like in the one dimensional case, you can consider the addition of more than three relative velocities to obtain the velocity of an object relative to you or to any other reference frame.

For example, consider a person walking on the boat with velocity $\vec{v}_{PB}$ relative to the boat. His velocity relative to the shore will be given by:

$$\vec{v}_{PS} = \vec{v}_{PB} + \vec{v}_{BW} + \vec{v}_{WS}$$

recalling previous slide:

$$\vec{v}_{BS} = \vec{v}_{BW} + \vec{v}_{WS} \quad \text{(3.19)}$$

Then

$$\text{(Eq. 3.19)} \quad \vec{v}_{PS} = \vec{v}_{PB} + \vec{v}_{BS}$$
Relative Velocity

**Notes:**

In general, if $\vec{v}_{AB}$ is the velocity of $A$ relative to $B$, then the velocity of $B$ relative to $A$, $\vec{v}_{BA}$, will be in the same line as $\vec{v}_{AB}$ but in opposite direction:

$$\vec{v}_{BA} = -\vec{v}_{AB} \quad (3.20)$$

If $\vec{v}_{CA}$ is the velocity of $C$ relative to $A$, then $\vec{v}_{CB}$, the velocity of $C$ relative to $B$, will be given by:

$$\vec{v}_{CB} = \vec{v}_{CA} + \vec{v}_{AB} \quad (3.21)$$
Relative Velocity

Problem 3.37 (textbook) : Huck Finn walks at a speed of 0.60 m/s across his raft (that is, he walks perpendicular to the raft’s motion relative to the shore). The raft is travelling down the Mississippi River at a speed of 1.70 m/s relative to the river bank (Fig. 3–38). What is Huck’s velocity (speed and direction) relative to the river bank?
Relative Velocity

37. Call the direction of the flow of the river the $x$ direction, and the direction of Huck walking relative to the raft the $y$ direction.

\[
\vec{v}_{\text{Huck rel. bank}} = \vec{v}_{\text{Huck rel. raft}} + \vec{v}_{\text{raft rel. bank}} = (0, 0.60) \text{ m/s} + (1.70, 0) \text{ m/s}
\]

\[
= (1.70, 0.60) \text{ m/s}
\]

Magnitude: \( v_{\text{Huck rel. bank}} = \sqrt{1.70^2 + 0.60^2} = 1.80 \text{ m/s} \)

Direction: \( \theta = \tan^{-1} \frac{0.60}{1.70} = 19^\circ \text{ relative to river} \)
Relative Velocity

**Problem 3-50 (textbook):** An unmarked police car, traveling a constant 95 Km/h is passed by a speeder traveling 145 Km/h. Precisely 1.00 s after the speeder passes, the policeman steps on the accelerator. If the police car’s acceleration is 2.00 m/s\(^2\): How much time elapses after the police car is passed until it overtakes the speeder (assumed moving at constant speed)?

*Solution developed on the blackboard*
3-50. Take the origin as the location at which the speeder passes the police car (in the reference frame of the unaccelerated police car).

The speeder is traveling at

\[
145 \text{ km/h} \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}}\right) = 40.28 \text{ m/s}
\]

relative to the ground, and the policeman is traveling at

\[
95 \text{ km/h} \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}}\right) = 26.39 \text{ m/s}
\]

relative to the ground.

Relative to the unaccelerated police car, the speeder is traveling at

\[
13.89 \text{ m/s} = v_s
\]

and the police car is not moving.
Relative Velocity

Do all calculations in the frame of reference of the unaccelerated police car:

The position of the speeder in the chosen reference frame is given by

\[ \Delta x_s = v_s t \]

The position of the (accelerated) policeman in the chosen reference frame is given by

\[ \Delta x_p = \frac{1}{2} a_p (t - 1)^2, \quad t > 1 \]

The police car overtakes the speeder when these two distances are the same.; i.e.:

\[ \Delta x_s = \Delta x_p \]
Relative Velocity

Then,

\[
\Delta x_s = \Delta x_p \quad \Rightarrow \quad v_s t = \frac{1}{2} a_p (t - 1)^2 \quad \Rightarrow \quad (13.89 \text{ m/s}) t = \frac{1}{2} \left( 2 \text{ m/s}^2 \right) (t^2 - 2t + 1) = t^2 - 2t + 1
\]

\[
t^2 - 15.89t + 1 = 0 \quad \Rightarrow \quad t = \frac{15.89 \pm \sqrt{15.89^2 - 4}}{2} = 0.0632 \text{ s}, 15.83 \text{ s}
\]

Since the police car doesn’t accelerate until \( t = 1.00 \text{ s} \), the correct answer is

\[
t = 15.8 \text{ s}
\]
• Backup Slides (supposedly for lecture-8)
We have introduced the concept of motion and obtained a set of equations that describe the motion of an object given some initial conditions. We have also introduced the concept of vector and have shown that velocity and acceleration are vectors (have magnitude and direction).

But.. we have not discussed what set an object in motion, or change its state of motion. This is what we will be learning in this chapter.
**Force**

The area that studies the connection between motion and its cause is called **dynamics**.

The cause of the motion is **force**.

But what force is??

**Force is the interaction between two objects.**

For example: When you push a grocery cart which is initially at rest, you are exerting a force on it.

**The force you apply on the cart changes its state of motion**
Force

Force is the interaction between two objects.

Side note:

You may ask me: **but how do two objects interact?**

**For example:** You have seen that magnets attract some materials such as a piece of iron. But the magnet does not need to touch the iron to attract it. So, how do these two objects interact?

The answer to this question requires advanced physics. However, let’s say that there are some special particles that operates as messengers between the two objects. These messengers are the carries of the force between these objects and therefore responsible for their interaction.

There are four fundamental kinds of forces:

- Gravitational  →  Responsible for having the Moon orbiting the Earth….
- Electromagnetic  →  Responsible for the attraction between the magnet and iron.
- Weak  →  Main responsible for the processes that make stars to shine
- Strong  →  Responsible for the nuclear interaction
→ Note that the direction of the grocery cart depends on the direction of the force you apply on it:

You can push it on a straight line, or to the left, or to the right, etc..

→ The strength you push the cart will determine how fast the cart goes or how fast you will change its direction.

So, it is clear that you should assign not only a direction, but also a magnitude to the force you apply on the cart. In other words,

Force is a vector.

And we will represent it using the standard vector representation: \( \vec{F} \) with the arrow representing the direction a force is applied on an object.