Chapter 3
Kinematics in Two Dimensions; Vectors
Vectors and Scalars

Addition of Vectors ï Graphical Methods (One and Two-Dimension)

Multiplication of a Vector by a Scalar

Subtraction of Vectors ï Graphical Methods

Adding Vectors by Components

Projectile Motion

Projectile Motion Is Parabolic

Relative Velocity
Short review with few questions – Chapter 2
You and your dog go for a walk to the park. On the way, your dog takes many side trips to chase a squirrel. When you arrive at the park, do you and your dog have the same displacement?

1) yes
2) no

Yes, you have the same displacement. Since you and your dog had the same initial and final positions, then you have (by definition) the same displacement.
2.2 Displacement

Does the displacement of an object depend on the specific location of the origin of the coordinate system?

1) yes
2) no
3) it depends on the coordinate system

Since the displacement is the difference between two coordinates, the origin does not matter.
If the position of a car is zero, does its speed have to be zero?

1) yes
2) no
3) it depends on the position

No, the speed does not depend on position, it depends on the change of position. Since we know that the displacement does not depend on the origin of the coordinate system, an object can easily start at $x = -3$ and be moving by the time it gets to $x = 0$. 
If the average velocity is non-zero over some time interval, does this mean that the instantaneous velocity is never zero during the same interval?

1) yes  
2) no  
3) it depends

No!!! For example, your average velocity for a trip home might be 60 mph, but if you stopped for lunch on the way home, there was an interval when your instantaneous velocity was zero, in fact!
Sure it can! An object moving with constant velocity has a non-zero velocity, but it has zero acceleration since the velocity is not changing.
Both balls are in free fall once they are released, therefore they both feel the acceleration due to gravity ($g$). This acceleration is independent of the initial velocity of the ball.

Which one has the greater velocity when they hit the ground?

1. Alice’s ball
2. it depends on how hard the ball was thrown
3. neither -- they both have the same acceleration
4. Bill’s ball
We have already introduced the concept of vectors and scalars in chapter 2. Let's now see some of the vector properties in more details.

Recalling:

**Scalar**: quantity specified by a number and some unit (Ex.: temperature, mass, etc)

**Vector**: entity that contains information about the magnitude and direction of certain quantity.

In our example, $\Delta x$ is the displacement vector pointing in the $+x$ direction with magnitude represented by $\Delta x$:

$$\Delta x = x_2 - x_1 \quad (2.1)$$

**Notes**: 1) The arrow is always drawn such that it points in the direction of the vector it represents.
   
   2) The magnitude of a vector is always positive. The sign in (2.1) gives the direction of the vector (Ex.: negative indicates that it points in the $-x$ direction).
Vectors and Scalars

The length of the arrow representing a vector is usually drawn proportional to the magnitude of the vector.
Addition of Vectors – Graphical Method

It is clear the importance of vectors when addressing problems in physics. We have already seen how important it is to understand motion in one dimension of an object by representing it using the displacement, velocity and acceleration vectors.

It is now useful to see some vector properties and understand how they can be used to solve physics problems not only in one dimension but also in two (or three) dimensions.

**Example 3.1 (one-dimension):**

(a) A person walks 8 Km in the +x direction, and then another 6 Km in the same direction (figure (a)). Use the vector method to obtain the person’s displacement.

(b) Solve the problem using figure (b).
Addition of Vectors – Graphical Method

(a) A person walks 8 Km in the +x direction, and then another 6 Km in the same direction (figure (a)). Use the vector method to obtain the person’s displacement.

Let $\vec{D}_1$ represent the first displacement (8 Km) by the vector $\vec{D}_1$; the second displacement (6 km) by $\vec{D}_2$; and the resulting displacement by $\vec{D}_R$.

We can say the following:

$$\vec{D}_R = \vec{D}_1 + \vec{D}_2$$

The magnitude will be given by

$$D_R = D_1 + D_2 = 8 \text{ Km} + 6 \text{ Km} = 14 \text{ Km}$$
Addition of Vectors – Graphical Method

(b) Solve the problem using figure (b).

Vector $\vec{D}_1$ is pointing in the $+x$ direction, but vector $\vec{D}_2$ points in the $-x$ direction.

The resulting displacement vector $\vec{D}_R$ can still be given by

$$\vec{D}_R = \vec{D}_1 + \vec{D}_2$$

But now we should be careful with the relative directions of the two vectors when calculating the resulting magnitude of $\vec{D}_R$.

We can consider $\vec{D}_2$ to be the opposite of a vector $\vec{D}$ of same magnitude as $\vec{D}_2$ such that:

$$\vec{D}_2 = -\vec{D}$$

$\vec{D}$ is said to be the negative of the vector $\vec{D}_2$. 

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Addition of Vectors – Graphical Method

We can then rewrite $\overrightarrow{D_R} = \overrightarrow{D_1} + \overrightarrow{D_2}$ as

$$\overrightarrow{D_R} = \overrightarrow{D_1} + \overrightarrow{D_2} = \overrightarrow{D_1} + (-\overrightarrow{D})$$

The magnitude will be given by:

$$\overrightarrow{D_R} = \overrightarrow{D_1} + (-\overrightarrow{D}) = 8 \text{ Km} + (-6 \text{ Km}) = 2 \text{ Km}$$

Note that the sign is that of the vector with greatest magnitude; $\overrightarrow{D_1}$ in this case. Thus, $\overrightarrow{D_R}$ will point in the $\overrightarrow{D_1}$ direction ($+x$) with a magnitude of 2 Km as depicted in figure (b).
Addition of Vectors – Graphical Method

The equations for vector addition obtained on the previous slides are valid for any arbitrary vectors. For instance, let \( \vec{V}_1 \) and \( \vec{V}_2 \) be any arbitrary vectors and \( \vec{V} \) the result of the addition of these two vectors. Then:

\[
\vec{V} = \vec{V}_1 + \vec{V}_2
\]

(3.1)

Will have the same properties as those discussed in the previous example:

The sign of the vector \( \vec{V} \) will be that of one of the addition vectors, \( \vec{V}_1 \) or \( \vec{V}_2 \), with the greatest magnitude.

**Note:** If you add more than three vectors, you can always effectuate the addition in steps. For example:

\[
\vec{V} = \vec{V}_1 + \vec{V}_2 + \vec{V}_3
\]

Let \( \vec{V}_4 \) be such that

\[
\vec{V}_4 = \vec{V}_2 + \vec{V}_3
\]

Then

\[
\vec{V} = \vec{V}_1 + \vec{V}_4
\]
Addition of Vectors – Graphical Method

Let $c$ be a scalar and $\vec{V}_1$ an arbitrary vector. The product of $c$ by $\vec{V}_1$ is a new vector $\vec{V}$ with magnitude $cV_1$. The direction of $\vec{V}$ is such that:

a) It points in the $\vec{V}_1$ direction if $c > 0$;

b) It points in the opposite direction of $\vec{V}_1$ if $c < 0$;

\[ \vec{V}_2 = 1.5 \vec{V} \]
\[ \vec{V}_3 = -2.0 \vec{V} \]
Addition of Vectors – Graphical Method

The discussions have been so far carried out based on one-dimensional space. But do the results obtained also apply in two-dimensional cases?

**Example 3.2:**

A person walks 10 Km from the origin 0 in the +x direction (eastward). He then decide to head north (±y direction) and walks another 5 Km. Obtain his displacement (direction and magnitude).

**Solution:**

Let \( \vec{D}_1 \) and \( \vec{D}_2 \) be the vectors representing his motion in the +x and +y directions, respectively. We know that his resulting displacement corresponds to the length between his initial and final positions (0 and B).

It is represented by the vector \( \vec{D}_R \) connecting these two positions and pointing from 0 to B.

As in the previous examples, the resulting displacement is the addition of the two intermediary displacements \( \vec{D}_1 \) and \( \vec{D}_2 \): \[ \vec{D}_R = \vec{D}_1 + \vec{D}_2 \]
Addition of Vectors – Graphical Method

However, a first look at the diagram below shows that in this case the magnitude of the resulting displacement is such that:

\[ D_R \neq D_1 + D_2 \]

if the vectors \( \vec{D}_1 \) and \( \vec{D}_2 \) are not along the same line.

In fact \( \vec{D}_R, \vec{D}_1 \) and \( \vec{D}_2 \) form a right triangle with \( D_R \) as the hypotenuse.
We can then use the theorem of Pythagoras to obtain \( D_R \):

\[
(\overrightarrow{OB})^2 = (\overrightarrow{OA})^2 + (\overrightarrow{AB})^2 \quad \text{yields} \quad D_r^2 = D_1^2 + D_2^2
\]

\[ D_R = \sqrt{D_1^2 + D_2^2} = \sqrt{(10 \text{ Km})^2 + (5 \text{ Km})^2} = \sqrt{125 \text{ Km}^2} = 11.2 \text{ Km} \]

The angle \( \theta \) can be measured using a protractor for example.
Addition of Vectors – Graphical Method

Based on the previous example, we can introduce the general rules for graphically adding two vectors, not matter the angles they make. This method is called tail-to-tip.

a) Choose a reference frame represented by a pair of coordinate axes;

b) Draw one of the vectors, say $\vec{D}_1$, to scale on your coordinate system. You can have this vector starting at the origin of your coordinate system if you want;

c) Draw the second vector, say $\vec{D}_2$, to scale, with its tail at the tip of the first vector;

d) The resultant, $\vec{D}_R$, is the vector connecting the tail of the first vector and the tip of the second vector:

$$\vec{D}_R = \vec{D}_1 + \vec{D}_2$$

It makes an angle $\theta$ with the $x$ axis. The length of $\vec{D}_R$ represents its magnitude.
Addition of Vectors – Graphical Method

**Note:**

The following identity is true:

\[ \vec{V} = \vec{V}_1 + \vec{V}_2 = \vec{V}_2 + \vec{V}_1 \]

(3.2)

It is not important in which order the vectors are added.
Addition of Vectors – Graphical Method

The tail-to-tip vector addition method is valid not only for vectors at right angle, but also for vectors at any angle:

\[ \vec{v} = \vec{v}_1 + \vec{v}_2 \]

The method can also be extended to three or more vectors.

**Note:** You can use the method of the vector addition step by step as mentioned before, and reduce the problem to the addition of two vectors if you want.
Addition of Vectors – Graphical Method

And alternative to the tail-to-tip method is the so called **parallelogram method**. In this method the following rules apply:

a) Two vectors, say \( \vec{V}_1 \) and \( \vec{V}_2 \), are drawn such the they start from a common origin chosen by you when you select you coordinate system;

b) A parallelogram is constructed using these two vectors as adjacent sides;

c) The resultant vector is diagonal drawn from the common origin.
Subtraction of Vectors – Graphical Method

We have actually discussed about subtraction of vectors when we were talking about addition of vectors. In fact, subtraction of vectors is the same as addition if you consider the following identity:

$$\vec{V} = \vec{V}_1 - \vec{V}_2 = \vec{V}_1 + (-\vec{V}_2)$$  \hspace{1cm} (3.3)

And all properties for addition applies for subtraction.

**Example 3.3:**

$$\vec{V}_2 - \vec{V}_1 = \vec{V}_2 + (-\vec{V}_1) = \vec{V}_2 - \vec{V}_1$$

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Vectors – Examples

**Problem 3.3 (textbook)**

Show that the vector labeled "incorrect" in Fig. 3-6c is actually the difference of the two vectors. Is it \( \vec{V}_2 - \vec{V}_1 \), or \( \vec{V}_1 - \vec{V}_2 \) ?

**Solution:**

Label the "INCORRECT" vector as vector \( \vec{X} \). Then Fig. 3-6 (c) illustrates the relationship \( \vec{V}_1 + \vec{X} = \vec{V}_2 \) via the tail-to-tip method. Thus

\[
\vec{X} = \vec{V}_2 - \vec{V}_1
\]
LAB

Â The revised schedule is now posted on the WEB and section 151 now has their first lab this Friday.

Shaun Szymanski.
Assignment 1

Textbook (Giancoli, 6th edition), Chapter 2. Due on Sept. 24th.


3. A sprinter, in the 100-m dash, accelerates from rest to a top speed with a (constant) acceleration of 2.80 m/s² and maintains the top speed to the end of the dash.

   (a) What time elapsed and

   (b) what distance did the sprinter cover during the acceleration phase if the total time taken in the dash was 12.2 s?

4. On page 41 of Giancoli, problem 44.

Please, note the Questions and Problems are two different things in the textbook. The assignment above includes only Problems.