Chapter 3

Kinematics in Two Dimensions; Vectors
Vectors and Scalars

Addition of Vectors — Graphical Methods (One and Two-Dimension)

Multiplication of a Vector by a Scalar

Subtraction of Vectors — Graphical Methods

Adding Vectors by Components

Projectile Motion

Projectile Motion Is Parabolic

Relative Velocity (Relative Motion)
(Re-)Recalling Last Two Lectures
Adding Vectors by Components

\[ \vec{V} = \vec{V}_x + \vec{V}_y \]

\[ (\vec{V})_x = V_x \]

\[ (\vec{V})_y = V_y \]

\[ V_x = V \cos \theta \]

\[ V_y = V \sin \theta \]

\[ |\vec{V}| = V = \sqrt{V_x^2 + V_y^2} \]

\[ \theta = \sin^{-1} \left( \frac{V_y}{V} \right) \]

\[ \theta = \tan^{-1} \left( \frac{V_y}{V_x} \right) \]

\[ \theta = \cos^{-1} \left( \frac{V_x}{V} \right) \]

\[ V^2 = V_x^2 + V_y^2 \]
Adding Vectors by Components

Note also that in general:

If \( \vec{A} \) and \( \vec{B} \) are two arbitrary vectors and \( \vec{C} \) is the result of their addition, then:

\[
\vec{C} = \vec{A} + \vec{B}
\]

\[
C = |\vec{C}|
\]

\[
B = |\vec{B}|
\]

\[
A = |\vec{A}|
\]

\[
C^2 = \sqrt{A^2 + B^2}
\]
Adding Vectors by Components

Representing a vector by its components:

We can define a vector using the following representation:

\[ \mathbf{v} = (v_x, v_y) \]  \hspace{1cm} (3.17)

Where in general:

\[(a, b) = (x \text{ component}, \ y \text{ component})\]

Example:

\[ \mathbf{c} = (-1.0, 5.0) \]

Tell me that

\[ c_x = -1.0 \]

\[ c_y = 5.0 \]
Adding Vectors by Components

More Comments:

If \( \vec{C} \) is a vector and \( C \) a scalar (for example, can be the vector magnitude), **the it is wrong to write**

\[ \vec{C} = C \]

Example:

\[ \vec{C} = 43.0 \]
Adding Vectors by Components

Note about arc-tan (\(\tan^{-1}\)):  

The sign of \(\theta\) in the expression below depends on the signs of \(V_y\) and \(V_x\):

\[
\theta = \tan^{-1}\left(\frac{V_y}{V_x}\right)
\]

\(\Delta \theta < 0\) if: \(V_y < 0\) and \(V_x > 0\), or \(V_y > 0\) and \(V_x < 0\)

\(\Delta \theta > 0\) if: \(V_y < 0\) and \(V_x < 0\), or \(V_y > 0\) and \(V_x > 0\)

The quadrant where the vector is defined is given by its coordinates:

\(\Delta V_y < 0\) and \(V_x > 0\) \(\rightarrow\) 4th quadrant
\(\Delta V_y < 0\) and \(V_x < 0\) \(\rightarrow\) 3rd quadrant
\(\Delta V_y > 0\) and \(V_x < 0\) \(\rightarrow\) 2nd quadrant
\(\Delta V_y > 0\) and \(V_x > 0\) \(\rightarrow\) 1st quadrant
**Projectile Motion**

We can then analyze the motion in the $x$ and $y$ direction separately using the equations of motion we obtained in chapter 2 (only vertical acceleration $g$ in $y$ direction):

- **Horizontal Motion**
  \[
  \begin{align*}
  v_x &= v_{x0} + at \\
  x &= x_0 + v_{x0}t + \frac{1}{2}a_xt^2 \\
  v_x^2 &= v_{x0}^2 + 2a_x(x - x_0)
  \end{align*}
  \]

- **Vertical Motion**
  \[
  \begin{align*}
  v_y &= v_{y0} + gt \\
  y &= y_0 + v_{y0}t + \frac{1}{2}gt^2 \\
  v_y^2 &= v_{y0}^2 + 2g(y - y_0)
  \end{align*}
  \]

**Note:** The sign of the acceleration $g$ in the above Eqs. depends on your choice of orientation for the axis $y$. You will have to rewrite the above equations using $-g$ instead of $g$ if you decide to choose $+y$ pointing upwards as in the figure.
Projectile Motion

If an object is **launched** at an initial angle of $\theta_0$ with the horizontal, the analysis is similar except that the initial velocity has a **vertical** component.

You can use equations the equations from previous slide to develop your problem in the very same way as when $\theta_0 = 0$.

$$V_{x0} = V_0 \cos \theta_0$$
$$V_{y0} = V_0 \sin \theta_0$$

$\vec{v}_y = 0$ at this point

$\vec{a} = \vec{g}$

$$V_x = V \cos \theta \quad (3.11)$$
$$V_y = V \sin \theta \quad (3.12)$$
Important Note

You might consider a problem where an object is subject to an acceleration other than that of gravity. The problem is again solved using the method of components where now the acceleration vector can also be represented by its components on the x and y axes:

In this particular case:

\[ \vec{a} = \vec{a}_x + \vec{a}_y \quad ; \quad |\vec{a}| = a \quad ; \quad (\vec{a})_x = a_x \quad ; \quad (\vec{a})_y = a_y \]

\[ a_x = a \cos \theta \quad ; \quad a_y = a \sin \theta \]

The resultant acceleration is the addition of \( \vec{a} \) and \( \vec{g} \):

\[ \vec{A} = \vec{a} + \vec{g} \]

\[ \vec{A}_y = \vec{a}_y + \vec{g} \quad \quad \vec{A}_x = \vec{a}_x \]

\[ (\vec{A})_x = (\vec{a})_x + (\vec{g})_x = a_x + 0 = a_x \]

\[ (\vec{A})_y = (\vec{a})_y + (\vec{g})_y = a_y - g \]
**Important Note**

The equations of motion in x and y are then written as:

**horizontal motion**

\[
\begin{align*}
v_x &= v_{x0} + a_x t \\
x &= x_0 + v_{x0} t + \frac{1}{2} a_x t^2 \\
v^2_x &= v_{x0}^2 + 2a_x(x-x_0)
\end{align*}
\]

**vertical motion**

\[
\begin{align*}
v_y &= v_{y0} + (a_y - g)t \\
y &= y_0 + v_{y0} t + \frac{1}{2} (a_y - g)t^2 \\
y-y_0 &= v_{y0} t + \frac{1}{2} (a_y - g)t^2
\end{align*}
\]

Note that the minus sign in \((\vec{a})_y = (\vec{a})_y + (\vec{g})_y = a_y - g\) is due to the choice of having the y axis pointing upwards.
It is not difficult to show that whenever we assume the acceleration of gravity to be constant and ignore the air resistance, then the vertical displacement $y$ can be written as a function of the horizontal displacement $x$. Using the equations of motion 2.10, 2.12, 2.13 and 2.16-2.18, we can show that:

$$y = Ax - Bx^2$$

I leave for you to obtain the values of $A$ and $B$ in terms of the initial angle $\theta_0$, initial velocity $v_0$ and acceleration of gravity $g$ (assume $y$ axis pointing upwards).
Relative Velocity

We have seen that it is important to choose a reference frame based on which we can define quantities such as velocity, position, etc.

An example was that of a person walking along a train which is moving relative to the ground. The velocity YOU measure of this person depends on whether you are on the train or standing on the ground.

It is common to place yourself at the origin of your coordinate system (reference frame) when you are the one performing the measurement.
Relative Velocity

This is an example of relative velocity. If the train moves with velocity \( \vec{v}_{TG} \) relative to the ground, and the person moves with velocity \( \vec{v}_{PT} \) relative to the train, then, you will measure the velocity \( \vec{v}_{PY} \) relative to you as:

\[
\vec{v}_{PY} = \vec{v}_{PT} + \vec{v}_{TY} = \vec{v}_{PT} + 0 = \vec{v}_{PT}
\]

Where \( \vec{v}_{TY} \) is the velocity of the train relative to you (zero in this case)

(ground is moving this way relative to you)
Relative Velocity

You are on the ground:

\[ \vec{v}_{PY} = \vec{v}_{PT} + \vec{v}_{TY} = \vec{v}_{PT} + \vec{v}_{TG} \]
Relative Velocity

In a more general case, you could be moving in a car with velocity $\vec{v}_{YG}$ relative to the ground. In this case you will measure:

First note that: $\vec{v}_{YG} = -\vec{v}_{GY}$ \hfill (3.17)

Then: $\vec{v}_{PY} = \vec{v}_{PT} + \vec{v}_{TG} + \vec{v}_{GY}$, or $\vec{v}_{PY} = \vec{v}_{PT} + \vec{v}_{TG} - \vec{v}_{YG}$ \hfill (3.18)
Relative Velocity

The previous example addressed a one dimensional problem. However, it can be easily generalized to two (or three) dimensional problems thanks to the fact that we have been using vectors as a method (tool) to analyze motion.

Let $\vec{v}_{BW}$ be the velocity of a boat relative to the river water and $\vec{v}_{WS}$ the velocity of the water relative to the shore. Then the velocity $\vec{v}_{BS}$ of the boat relative to the shore will be given by:

$$\vec{v}_{BS} = \vec{v}_{BW} + \vec{v}_{WS} \quad (3.19)$$

You can then use the component method for vector addition to obtain $\vec{v}_{BS}$.
Relative Velocity

Note that like in the one dimensional case, you can consider the addition of more than three relative velocities to obtain the velocity of an object relative to you or to any other reference frame.

For example, consider a person walking on the boat with velocity $\vec{v}_{PB}$ relative to the boat. His velocity relative to the shore will be given by:

$$\vec{v}_{PS} = \vec{v}_{PB} + \vec{v}_{BW} + \vec{v}_{WS}$$

recalling previous slide:

$$\vec{v}_{BS} = \vec{v}_{BW} + \vec{v}_{WS} \quad (3.19)$$

Then

$$\overset{(Eq. 3.19)}{\Rightarrow} \quad \vec{v}_{PS} = \vec{v}_{PB} + \vec{v}_{BS}$$
Relative Velocity

**Notes:**

In general, if \( \vec{v}_{AB} \) is the velocity of \( A \) relative to \( B \), then the velocity of \( B \) relative to \( A \), \( \vec{v}_{BA} \), will be in the same line as \( \vec{v}_{AB} \) but in opposite direction:

\[
\vec{v}_{BA} = -\vec{v}_{AB}
\] (3.20)

If \( \vec{v}_{CA} \) is the velocity of \( C \) relative to \( A \), then \( \vec{v}_{CB} \), the velocity of \( C \) relative to \( B \), will be given by:

\[
\vec{v}_{CB} = \vec{v}_{CA} + \vec{v}_{AB}
\] (3.21)
**Problem 3.37 (textbook)**: Huck Finn walks at a speed of 0.60 m/s across his raft (that is, he walks perpendicular to the raft's motion relative to the shore). The raft is traveling down the Mississippi River at a speed of 1.70 m/s relative to the river bank (Fig. 3i 38). What is Huck's velocity (speed and direction) relative to the river bank?
Relative Velocity

37. Call the direction of the flow of the river the $x$ direction, and the direction of Huck walking relative to the raft the $y$ direction.

\[
\vec{v}_{\text{Huck rel. bank}} = \vec{v}_{\text{Huck rel. raft}} + \vec{v}_{\text{raft rel. bank}} = (0, 0.60) \text{ m/s} + (1.70, 0) \text{ m/s}
\]

\[
= (1.70, 0.60) \text{ m/s}
\]

Magnitude: \( v_{\text{Huck rel. bank}} = \sqrt{1.70^2 + 0.60^2} = 1.80 \text{ m/s} \)

Direction: \( \theta = \tan^{-1} \frac{0.60}{1.70} = 19^\circ \text{ relative to river} \)
Relative Velocity

**Problem 5.50 (textbook)**: An unmarked police car, traveling a constant 95 Km/h is passed by a speeder traveling 145 Km/h. Precisely 1.00 s after the speeder passes, the policeman steps on the accelerator. If the police car's acceleration is 2.00 m/s$^2$, how much time elapses after the police car is passed until it overtakes the speeder (assumed moving at constant speed)?

Solution developed on the blackboard
52. Take the origin to be the location at which the speeder passes the police car, in the reference frame of the unaccelerated police car.

The speeder is traveling at

\[ 145 \text{ km/h} \left( \frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) = 40.28 \text{ m/s} \]

relative to the ground, and the policeman is traveling at

\[ 95 \text{ km/h} \left( \frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) = 26.39 \text{ m/s} \]

relative to the ground.

Relative to the unaccelerated police car, the speeder is traveling at

\[ 13.89 \text{ m/s} = v_s \]

and the police car is not moving.
Relative Velocity

Do all of the calculations in the frame of reference of the unaccelerated police car.

The position of the speeder in the chosen reference frame is given by

$$\Delta x_s = v_s t$$

The position of the policeman in the chosen reference frame is given by

$$\Delta x_p = \frac{1}{2} a_p \left(t - 1\right)^2, t > 1$$

The police car overtakes the speeder when these two distances are the same.; i.e.:

$$\Delta x_s = \Delta x_p$$
Relative Velocity

Then,

\[ \Delta x_s = \Delta x_p \rightarrow v_s t = \frac{1}{2} a_p (t - 1)^2 \rightarrow (13.89 \text{ m/s}) t = \frac{1}{2} (2 \text{ m/s}^2)(t^2 - 2t + 1) = t^2 - 2t + 1 \]

\[ t^2 - 15.89t + 1 = 0 \rightarrow t = \frac{15.89 \pm \sqrt{15.89^2 - 4}}{2} = 0.0632 \text{ s , } 15.83 \text{ s} \]

Since the police car doesn't accelerate until \( t = 1.00 \text{ s} \), the correct answer is

\[ t = 15.8 \text{ s} \]
Assignment 3

Textbook (Giancoli, 6th edition), Chapter 3:

Due on Thursday, October 2, 2008

- Problem 27 - page 67 of the textbook

- Problems 41 and 47 - page 68 of the textbook

- Problem 68 - page 71 of the textbook