Chapter 5 – Gravitation

Chapter 6 – Work and Energy
Chapter 5

• (5.6) Newton’s Law of Universal Gravitation
• (5.7) Gravity Near the Earth’s Surface

Chapter 6 (today)

• Work Done by a Constant Force
• Kinetic Energy, and the Work-Energy Principle
Old assignments can be picked up
in my office

(LB-212)
Recalling Last Lecture
Newton’s Law of Universal Gravitation

Newton’s law of universal gravitation:

“Every particle in the universe attracts every other particle with a force that is proportional to the products of their masses and inversely proportional to the square of the distance between them. This force acts along the line joining the two particles.”
Newton’s Law of Universal Gravitation

The magnitude of this force is given by:

$$F = G \frac{m_1 m_2}{r^2}$$  \hspace{1cm} (4.7)

Where:

$$m_1 = \text{mass of object 1}$$

$$m_2 = \text{mass of object 2}$$

$$r = \text{distance between the two objects}$$

$$G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{Kg}^2 \hspace{1cm} \text{is the universal gravitational constant}$$
Newton’s Law of Universal Gravitation

If \( m_2 \) is the object exerting a force on \( m_1 \), then the gravitational acceleration felt by object 1 can be identified as:

\[
F_1 = \left( G \frac{m_2}{r^2} \right) m_1 \Rightarrow F_1 = m_1 a_2 \Rightarrow a_2 = G \frac{m_2}{r^2} \tag{4.8}
\]

And vice-versa: object 2 will feel an acceleration

\[
a_1 = G \frac{m_1}{r^2}
\]

due to the gravitational force applied by object 1.

You can then write \( F_1 = m_1 a_2 \) for the force acting on \( m_1 \) and \( F_2 = m_2 a_1 \) for the force acting on \( m_2 \).
Gravity Near the Earth’s Surface

An object of mass \( m \) on the surface of the Earth will feel a force given by:

\[
F = mg = G \frac{mm_E}{r_E^2}
\]

(4.9)

Where,

\( m_E = \text{mass of the Earth} \);
\( r_E = 6.38 \times 10^6 \text{ m} = \text{radius of the Earth} \).
\( g \) has been measured and it is known to be 9.80 m/s\(^2\).

In fact, the value of \( g \) can be considered constant at any position near the Earth’s surface.

Accurate measurements yield: \( m_E = 5.974 \times 10^{24} \text{ Kg} \).
Newton’s Law of Universal Gravitation

Problem 5-31 (textbook): A hypothetical planet has a radius 1.5 times that of Earth, but has the same mass. What is the acceleration due to gravity near its surface?

Solution:

The acceleration due to gravity at any location on or above the surface of a planet is given by

\[ g_{\text{planet}} = \frac{G M_{\text{planet}}}{r^2} \]

where \( r \) is the distance from the center of the planet to the location in question.

\[ g_{\text{planet}} = G \frac{M_{\text{planet}}}{r^2} = G \frac{M_{\text{Earth}}}{(1.5 R_{\text{Earth}})^2} = \frac{1}{1.5^2} G \frac{M_{\text{Earth}}}{R_{\text{Earth}}^2} = \frac{1}{1.5^2} g_{\text{Earth}} = \frac{9.8 \text{ m/s}^2}{1.5^2} = 4.4 \text{ m/s}^2 \]
Newton’s Law of Universal Gravitation

Problem 5-34 (textbook): Calculate the effective value of \(g\), the acceleration of gravity, at (a) 3200 m, and (b) 3200 km, above the Earth’s surface.

Solution:

The acceleration due to gravity at any location at or above the surface of a planet is given by

\[
g_{\text{planet}} = G \frac{M_{\text{planet}}}{r^2}
\]

where \(r\) is the distance from the center of the planet to the location in question. For this problem, \(M_{\text{planet}} = M_{\text{Earth}} = 5.97 \times 10^{24} \text{ kg}\)

(a)

\[
r = R_{\text{Earth}} + 3200 \text{ m} = 6.38 \times 10^6 \text{ m} + 3200 \text{ m}
\]

\[
g = G \frac{M_{\text{Earth}}}{r^2} = \left(6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2\right) \frac{(5.97 \times 10^{24} \text{ kg})}{(6.38 \times 10^6 \text{ m} + 3200 \text{ m})^2} = 9.77 \text{ m/s}^2
\]

(b)

\[
r = R_{\text{Earth}} + 3200 \text{ km} = 6.38 \times 10^6 \text{ m} + 3.20 \times 10^6 \text{ m} = 9.58 \times 10^6 \text{ m}
\]

\[
g = G \frac{M_{\text{Earth}}}{r^2} = \left(6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2\right) \frac{(5.97 \times 10^{24} \text{ kg})}{(9.58 \times 10^6 \text{ m})^2} = 4.34 \text{ m/s}^2
\]
Chapter 6
Work Done by a Constant Force

So far, we have discussed the motion of an object in terms of Newton’s three laws of motion. However, there is a more powerful way to develop these studies using the concepts of momentum and energy.

Most theories in physics are based on the fact that the TOTAL energy and momentum of a system are conserved, or in other words, it does not change with time. These are known as conservation laws of energy and momentum.

In general, it is easier to analyze the motion of a body using these concepts rather than using Newton’s three laws.

In fact, the Newton’s laws are not even completely correct as they are just an approximation for objects moving at very low speeds compared to the speed of light (300.000 Km/s) → they have to be corrected to account for very fast moving objects.

On the other hand, the conservation laws are valid regardless how fast an object moves.
Work Done by a Constant Force

So, let’s get started!

The work done by a constant force is defined as the distance moved multiplied by the component of the force in the direction of displacement:

\[ W = Fd \cos \theta \]  

(6-1)
Work Done by a Constant Force

In the SI system, the units of work are joules:

\[ 1 \text{ J} = 1 \text{ N} \cdot \text{m} \]

This follows from equation 6.1:

\[ W = Fd \cos \theta \]

Units of force are Newtons;
Units of distance are meters.

\[ [W] = [N] \cdot [m] = [J] \]
Work Done by a Constant Force

**Note:**

Assume that this person is moving with constant velocity (no net force applied) along a horizontal plane.

Then, as long as the person does NOT lift or lower the bag of groceries, he is doing NO work on it. The force he exerts has NO component in the direction of motion.

He applies a force to counter-balance the force of gravity. Both forces make a 90° angle relative to the displacement of the bag. From eq. 6.1 follows:

\[ W_P = F_P d \cos \theta = F_P d \cos(90^\circ) = 0 \]

\[ W_G = F_G d \cos \theta = F_G d \cos(90^\circ) = 0 \]
Work Done by a Constant Force

Work is a scalar quantity → it has only magnitude that can be positive or negative.

→ It is positive when the component of the force along the line of displacement is in the direction of the displacement vector.

\[ \vec{F}_d = \vec{F} \cos \theta \]

→ It is negative when the opposite happen.

\[ \vec{F}_d = \vec{F} \cos \theta \]

Note that friction force always does negative work (it always points in the opposite direction relative to the displacement).
Problems 6-3 (textbook): A 1300-N crate rests on the floor. How much work is required to move it at constant speed

(a) 4.0 m along the floor against a friction force of 230 N, and

(b) 4.0 m vertically?
Work Done by a Constant Force

Problem 6-3:

(a)

See the free-body diagram for the crate as it is being pulled.

Since the crate is not accelerating horizontally,

\[ F_p = F_{fr} = 230 \text{ N} \]

The work done to move it across the floor is the work done by the pulling force. The angle between the pulling force and the direction of motion is \(0^\circ\).

\[ W_p = F_p d \cos 0^\circ = (230 \text{ N})(4.0 \text{ m})(1) = 9.2 \times 10^2 \text{ J} \]
Problem 6-3:

(b)

The diagram of forces for this problem is shown.

Since the crate is not accelerating vertically, the pulling force is the same magnitude as the weight. The angle between the pulling force and the direction of motion is 0°.

\[
W_p = F_p d \cos 0° = mgd = (1300 \text{ N})(4.0 \text{ m}) = 5.2 \times 10^3 \text{ J}
\]
Problem 6-10 (textbook): What is the minimum work needed to push a 950-kg car 810 m up along a 9.0° incline?

(a) Ignore friction.

(b) Assume the effective coefficient of friction retarding the car is 0.25.
Work Done by a Constant Force

Problem 6-10:

Draw a free-body diagram of the car on the incline. Include a frictional force, but ignore it in part (a) of the problem.

The minimum work will occur when the car is moved at a constant velocity.
Work Done by a Constant Force

Problem 6-10:

(a)

Write Newton’s 2\textsuperscript{nd} law in both the \(x\) and \(y\) directions, noting that the car is unaccelerated

\[
\sum F_y = F_N - mg \cos \theta = 0 \quad \rightarrow \quad F_N = mg \cos \theta
\]

\[
\sum F_x = F_p - mg \sin \theta = 0 \quad \rightarrow \quad F_p = mg \sin \theta
\]

The work done by \(\vec{F}_p\) in moving the car a distance \(d\) along the plane (parallel to \(\vec{F}_p\)) is given by

\[
W_p = F_p d \cos 0^\circ = mgd \sin \theta = (950 \text{ kg})\left(9.80 \text{ m/s}^2\right)(810 \text{ m}) \sin 9.0^\circ = \boxed{1.2 \times 10^6 \text{ J}}
\]
Work Done by a Constant Force

Problem 6-10:

(b)

Now include the frictional force, given by: \( F_{fr} = \mu_k F_N \)

We still assume that the car is not accelerated. We again write Newton’s 2\textsuperscript{nd} law for each direction:

→ The \( y \)-forces are unchanged by the addition of friction, and so we still have

\[ F_N = m g \cos \theta \]

→ In the \( x \)-direction we have

\[ \sum F_x = F_p - F_{fr} - mg \sin \theta = 0 \Rightarrow F_p = F_{fr} + mg \sin \theta = \mu_k mg \cos \theta + mg \sin \theta \]

The work done by \( \vec{F}_p \) in moving the car a distance \( d \) along the plane (parallel to \( \vec{F}_p \) ) is given by

\[ W_p = F_p d \cos 0^\circ = mgd \left( \sin \theta + \mu_k \cos \theta \right) \]

\[ = (950 \text{ kg})(9.80 \text{ m/s}^2)(810 \text{ m})\left( \sin 9.0^\circ + 0.25 \cos 9.0^\circ \right) = 3.0 \times 10^6 \text{ J} \]
I mentioned, in the beginning of this lecture, that the concept of energy is very important in physics. Yet, I started discussing about work done by a force.

→ What is the connection between these two quantities?

A good definition of energy was introduced by the Einstein’s theory of relativity:

\[ E = mc^2 \]

But this is beyond the scope of this course.

In mechanics, we can use a less precise definition of energy as:

“The ability to do work”

In this course we will be defining translational kinetic energy and some forms of potential energies, though other forms the energy exist such as nuclear energy, heat energy, etc.
Kinetic Energy, and the Work Energy Principle

**Kinetic Energy**

A moving object can do work on a second object. Example:

A car hits another car → the first car applies a force on the second (say, at rest) which consequently undergoes a displacement.

This has the implication that the first car has the ability to do work, consequently it has (or carries) energy.

(to be continued in the next lecture)